Capitolul 5. Integrala Riemann.

1. **Să se calculeze:**

1.1. $\displaystyle\int_{-1}^{2} x^2 \ dx.$

1.2. $\displaystyle\int_{-1}^{2} \sqrt[3]{x} \ dx.$

1.3. $\displaystyle\int_{-1}^{1} (4x^3 - 3x^2 + 2x - 1) \ dx.$

1.4. $\displaystyle\int_{1}^{3} (x^2 + x - 2) \ dx.$

1.5. $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin x \ dx.$

1.6. $\displaystyle\int_{0}^{\pi} \cos x \ dx.$

1.7. $\displaystyle\int_{0}^{\frac{\pi}{4}} \frac{x^2}{1 + x^2} \ dx.$

1.8. $\displaystyle\int_{0}^{1} \frac{dx}{\sqrt{x^2 + 1}}.$

1.9. $\displaystyle\int_{0}^{1} e^x \ dx.$

1.10. $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\cos^2 x}.$

1.11. $\displaystyle\int_{-\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{dx}{\sqrt{1 - x^2}}.$

1.12. $\displaystyle\int_{\frac{1}{e}}^{e^2} \frac{dx}{x}.$

1.13. $\displaystyle\int_{0}^{1} \frac{dx}{1 + x^2}.$

1.14. $\displaystyle\int_{0}^{\frac{\pi}{2}} x \sin x \ dx.$

1.15. $\displaystyle\int_{1}^{e} x \ln x \ dx.$

1.16. $\displaystyle\int_{2}^{3} \frac{x + 1}{x^2(x - 1)} \ dx.$

1.17. $\displaystyle\int_{1}^{2} \frac{e^{\frac{1}{x^2}}}{x^3} \ dx.$

1.18. $\displaystyle\int_{0}^{1} x e^x \ dx.$

1.19. $\displaystyle\int_{-\pi}^{\pi} \sin^2 x \; dx.$ 1.20. $\displaystyle\int_{-\pi}^{\pi} \cos^2 x \; dx.$

1.21. $\displaystyle\int_{e}^{e^3} \frac{dx}{x \ln x}.$ 1.22. $\displaystyle\int_{0}^{\frac{\pi}{4}} \text{tg}^3 x \; dx.$

2. Să se calculeze ariile plane limitate de curbele:

2.1. $f(x) = 3x - x^2, \quad g(x) = 0.$

2.2. $f(x) = 4x - x^2, \quad g(x) = 0.$

2.3. $f(x) = x^2 + 1, \quad g(x) = 2.$

2.4. $f(x) = x^2, \quad g(x) = 4.$

2.5. $f(x) = x^2, \quad g(x) = x + 2.$

2.6. $f(x) = x^2 - x, \quad g(x) = 3x.$

2.7. $f(x) = 2x - x^2, \quad g(x) = x.$

2.8. $f(x) = (x - 1)^2 + 2, \quad g(x) = 3x - 1.$

2.9. $f(x) = x^2, \quad g(x) = 2x - x^2.$

2.10. $f(x) = x^2, \quad g(x) = 3x + 4.$

2.11. $f(x) = x^3, \quad g(x) = \sqrt{x}.$

2.12. $f(x) = \dfrac{5}{x}, \quad g(x) = 6 - x.$

2.13. $f(x) = x^2, \quad g(x) = \sqrt[3]{x}.$

2.14. $f(x) = x^2, \quad g(x) = 2\sqrt{2x}.$

2.15. $f(x) = -\sqrt{x}, \quad g(x) = \sqrt{x}, \quad x \in [0,4].$

2.16. $f(x) = e^x, \quad g(x) = e^{-x}, \quad x \in [0,1].$

2.17. $f(x) = \ln x, \quad g(x) = \ln^2 x.$

2.18. $f(x) = \dfrac{1}{4}|4 - x^2|, \quad g(x) = 7 - |x|.$

2.19. $f(x) = 0, \quad g(x) = -x + 2, \quad h(x) = \sqrt{x}.$

2.20. $f(x) = \dfrac{1}{x}, \quad g(x) = x, \quad x = 2.$

2.21. $f(x) = \sin x, \quad g(x) = \cos x, \quad x \in \left[0, \dfrac{\pi}{4}\right].$

2.22. $f(x) = x - \dfrac{\pi}{2}, \quad g(x) = \cos x, \quad x = 0.$

2.23. $f(x) = \sin^2 x, \quad g(x) = x\sin x, \quad x \in [0, \pi].$

2.24. $f(x) = \sin 2x, \quad g(x) = \sin x, \quad x \in \left[\dfrac{\pi}{3}, \pi\right].$

2.25. $f(x) = \operatorname{tg} x, \quad g(x) = \dfrac{2}{3}\cos x, \quad x = 0.$

2.26. $f(x) = \arcsin x, \quad g(x) = \arccos x, \quad h(x) = 0.$

2.27. $f(x) = 2^{x-2} + 1, \quad g(x) = 2^{2-x} + 1, \quad h(x) = \dfrac{3}{2}.$

2.28. $f(x) = 2 - |2 - x|, \quad g(x) = \dfrac{6}{|x+1|}.$

2.29. $f(x) = \big|\lg x\big|, \quad g(x) = 0, \quad x = \dfrac{1}{10}, \quad x = 10.$

2.30. $f(x) = \ln|1 + x|, \quad g(x) = -xe^{-x}, \quad x = 1.$

3. Să se calculeze ariile plane limitate de curbele:

3.1. $\rho^2 = a^2 \cos 2\varphi$.

3.2. $x = a \cos t, \quad y = b \sin t$.

3.3. $\rho = 4 \sin^2 \varphi$.

3.4. $x = a \cos^3 t, \quad y = a \sin^3 t$.

3.5. $\rho = a(1 + \cos \varphi)$.

3.6. $x = \dfrac{c^2}{a} \cos^3 t, \quad y = \dfrac{c^2}{b} \sin^3 t, \quad c^2 = a^2 - b^2$.

3.7. $\rho = 2 + \cos \varphi$.

3.8. $x = \dfrac{1 - t^2}{(1 + t^2)^2}, \quad y = \dfrac{2at}{(a + t^2)^2}$.

3.9. $\rho = a \sin 2\varphi$.

3.10. $x = t - t^2, \quad y = t^2 - t^3$.

3.11. $\rho = a \cos \varphi, \quad \rho = a(\cos \varphi + \sin \varphi)$.

3.12. $x = t^2 - 1, \quad y = t^3 - t^2$.

3.13. $\rho = 2 - \cos \varphi, \quad \rho = \cos \varphi$.

3.14. $x = \dfrac{t - t^3}{1 + 3t^2}, \quad y = \dfrac{4t^2}{1 + 3t^2}$.

3.15. $\rho = 2\sqrt{3} \cos \varphi, \quad \rho = 2 \sin \varphi$.

3.16. $x = \sin 2t, \quad y = \sin t$.

3.17. $\rho = 1 + \sqrt{2} \cos \varphi$.

3.18. $x = 1 + t - t^3, \quad y = 1 - 15t^2$.

3.19. $\rho = 3\sin\varphi, \quad \rho = 5\sin\varphi.$

3.20. $x = 1 + 2\cos t, \quad y = \operatorname{tg} t + 2\sin t.$

4. Să se calculeze lungimile arcelor:

4.1. $\quad f(x) = \dfrac{(x+1)^2}{4} - \dfrac{\ln(x+1)}{2}, \quad x \in [0,1].$

4.2. $\quad f(x) = -\ln\cos x, \quad x \in \left[0, \dfrac{\pi}{6}\right].$

4.3. $\quad f(x) = \ln x, \quad x \in \left[\sqrt{3}, \sqrt{8}\right].$

4.4. $\quad f(x) = \ln(x^2 - 1), \quad x \in [2,3].$

4.5. $\quad f(x) = \sqrt{2x - x^2} - 1, \quad x \in \left[\dfrac{1}{4}, 1\right].$

4.6. $\quad f(x) = x^2, \quad x \in [0,1].$

4.7. $\quad f(x) = 4\sqrt{x-1}, \quad x \in [1,2].$

4.8. $\quad f(x) = x^2 - \ln\sqrt{x}, \quad x \in [1,2].$

4.9. $\quad f(x) = x\sqrt{x}, \quad x \in [0,9].$

4.10. $\quad f(x) = \ln\sin x, \quad x \in \left[\dfrac{\pi}{3}, \dfrac{2\pi}{3}\right].$

4.11. $\quad x = a\cos^3 t, \quad y = a\sin^3 t, \quad t \in [0, 2\pi].$

4.12. $\quad \rho = 2\sin\varphi.$

4.13. $\quad x = 3(2 - t^2), \quad y = 4t^3, \quad x \geqslant 0.$

4.14. $\quad \rho = \cos^3\dfrac{\varphi}{3}.$

4.15. $x = \cos^4 t, \quad y = \sin^4 t, \quad t \in \left[0, \dfrac{\pi}{2}\right].$

4.16. $\rho = a(1 - \cos\varphi).$

4.17. $x = 6\cos^3 t, \quad y = 6\sin^3 t, \quad t \in \left[0, \dfrac{\pi}{3}\right].$

4.18. $\rho = \sin 3\varphi.$

4.19. $x = 2(t - \sin t), \quad y = 2(1 - \cos t), \quad t \in \left[0, \dfrac{\pi}{2}\right].$

4.20. $\rho = \dfrac{1}{2} + \sin\varphi.$

4.21. $x = e^t(\cos t + \sin t), \quad y = e^t(\cos t - \sin t), \quad t \in \left[\dfrac{\pi}{6}, \dfrac{\pi}{4}\right].$

4.22. $\rho = \cos\varphi - \sin\varphi.$

4.23. $x = 2(\cos t + t\sin t), \quad y = 2(\sin t - t\cos t), \quad t \in [0, \pi].$

4.24. $\rho = 2\sin 4\varphi.$

5. **Să se calculeze volumul corpului obţinut prin rotaţia în jurul axei OX a suprafeţei mărginite de curbele:**

   5.1. $f(x) = -x^2 + 7x - 12, \quad g(x) = 0.$

   5.2. $f(x) = \dfrac{4}{x}, \quad g(x) = 0, \quad x = 1, \quad x = 4.$

   5.3. $f(x) = 2x + \sqrt{2x}, \quad g(x) = 0, \quad x = 2 \quad x = \dfrac{9}{2}.$

   5.4. $f(x) = 2x - x^2, \quad g(x) = 2 - x.$

5.5. $f(x) = \arcsin x$, $\quad x = 0$, $\quad x = 1$.

5.6. $f(x) = xe^x$, $\quad g(x) = 0$, $\quad x = 1$.

5.7. $f(x) = x^2$, $\quad g(x) = 0$, $\quad x = 3$.

5.8. $f(x) = (x - 2)^2$, $\quad g(x) = 4$.

5.9. $f(x) = e^{2-x}$, $\quad g(x) = 0$, $\quad x = 1$, $\quad x = 2$.

5.10. $f(x) = e^x$, $\quad g(x) = 0$, $\quad x = 0$, $\quad x = 1$.

5.11. $f(x) = 3\sin x$, $\quad g(x) = \sin x$, $\quad x = 0$, $\quad x = \pi$.

5.12. $f(x) = \sin x$, $\quad g(x) = 0$, $\quad x = \dfrac{\pi}{6}$, $\quad x = \dfrac{\pi}{2}$.

5.13. $f(x) = 4 - x^2$, $\quad g(x) = 3x$, $\quad x = -2$, $\quad x = 0$.

5.14. $f(x) = \sqrt{x}\,e^{-x}$, $\quad g(x) = 0$, $\quad x = 1$.

5.15. $f(x) = \sin^2 x$, $\quad g(x) = x\sin x$, $\quad x = 0$, $\quad x = \pi$.

5.16. $f(x) = \sin 2x$, $\quad g(x) = 0$, $\quad x = 0$, $\quad x = \dfrac{\pi}{4}$.

5.17. $f(x) = 3x - x^2$, $\quad g(x) = 0$.

5.18. $x = a\cos^3 t$, $\quad y = a\sin^3 t$.

5.19. $f(x) = x^2$, $\quad g(x) = \sqrt{x}$.

5.20. $f(x) = x^3$, $\quad g(x) = x^2$.