

Capitolul 3. Derivabilitate.

1. Să se calculeze derivata funcției:

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|---|--|
| 1.1. $f(x) = x^3 + x^2 - x + 1.$ | 1.2. $f(x) = \frac{1}{4} x^4 - \frac{1}{3} x^3 + 2x^2 - 1.$ |
| 1.3. $f(x) = 2x^5 - x^{-2} + 3x.$ | 1.4. $f(x) = \frac{1}{x} - \frac{4}{x^2} - \frac{1}{x^3} + x.$ |
| 1.5. $f(x) = x^{\frac{1}{2}} + x^{\frac{2}{3}} - x^{-\frac{1}{3}}.$ | 1.6. $f(x) = \sqrt[3]{x} - \frac{1}{\sqrt[3]{x}} + 1.$ |
| 1.7. $f(x) = e^x \sin x.$ | 1.8. $f(x) = \operatorname{tg} x \ln x.$ |
| 1.9. $f(x) = 2^x \operatorname{ctg} x.$ | 1.10. $f(x) = x \arcsin x.$ |
| 1.11. $f(x) = (x^2 + 1) \operatorname{arctg} x.$ | 1.12. $f(x) = x^2 \ln x.$ |
| 1.13. $f(x) = \cos x \ln x.$ | 1.14. $f(x) = x \operatorname{arcctg} x.$ |
| 1.15. $f(x) = \frac{x}{x^2 - 1}.$ | 1.16. $f(x) = \frac{x^2 - 1}{x^2 + 1}.$ |
| 1.17. $f(x) = \frac{\sin x}{\ln x}.$ | 1.18. $f(x) = \frac{\operatorname{arctg} x}{e^x}.$ |
| 1.19. $f(x) = \frac{\sin x - \cos x}{\sin x + \cos x}.$ | 1.20. $f(x) = \frac{1 - \sin x}{1 + \sin x}.$ |
| 1.21. $f(x) = \ln 3 - \cos 2.$ | 1.22. $f(x) = \arcsin x + \arccos x.$ |

2. Să se calculeze derivata funcției:

2.1. $f(x) = (x^2 + 1)^{10}$. 2.2. $f(x) = \frac{1}{(x^2 + 2x + 3)^3}$.

2.3. $f(x) = \sqrt{x^2 - x + 7}$. 2.4. $f(x) = \frac{1}{\sqrt[3]{x^3 + x^2 + 1}}$.

2.5. $f(x) = \sin^2 x$. 2.6. $f(x) = \ln^2 x$.

2.7. $f(x) = \sin 3x$. 2.8. $f(x) = \sin(\ln x)$.

2.9. $f(x) = \cos 2x$. 2.10. $f(x) = \cos(e^x)$.

2.11. $f(x) = \operatorname{tg} 3x$. 2.12. $f(x) = \operatorname{tg} 2^x$.

2.13. $f(x) = \operatorname{ctg} \frac{x}{2}$. 2.14. $f(x) = \operatorname{ctg}(x^2 + x + 1)$.

2.15. $f(x) = e^{\sin x}$. 2.16. $f(x) = e^{-x}$.

2.17. $f(x) = 2^{\operatorname{tg} x}$. 2.18. $f(x) = 3^{\sqrt{x}}$.

2.19. $f(x) = \ln(\sin x)$. 2.20. $f(x) = \ln(\operatorname{arctg} x)$.

2.21. $f(x) = \operatorname{arctg} \sqrt{x}$. 2.22. $f(x) = \operatorname{arctg} e^x$.

2.23. $f(x) = \operatorname{arcsin} \sqrt{x}$. 2.24. $f(x) = \operatorname{arcsin} e^{-x}$.

3. Să se calculeze derivata funcției:

3.1. $f(x) = \ln \operatorname{tg} \frac{x}{2}$. 3.2. $f(x) = \ln \left(x + \sqrt{x^2 + 1} \right)$.

3.3. $f(x) = \ln \sqrt[4]{\frac{1 - \sin x}{1 + \sin x}}$. 3.4. $f(x) = \ln \frac{x^2 - 1}{x^2 + 1}$.

3.5. $f(x) = \ln \sin \frac{2x + 4}{x + 1}$. 3.6. $f(x) = \ln \operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{4} \right)$.

3.7. $f(x) = \operatorname{arctg} \sqrt{4x - 1}$. 3.8. $f(x) = \sqrt{x} - \operatorname{arctg} \sqrt{x}$.

3.9. $f(x) = \operatorname{arctg} \frac{1 + x}{1 - x}$. 3.10. $f(x) = \operatorname{arctg} \frac{x}{1 + \sqrt{1 + x^2}}$.

3.11. $f(x) = \arcsin \frac{1 - x}{\sqrt{2}}$. 3.12. $f(x) = \arcsin \sqrt{1 - x^2}$.

3.13. $f(x) = \arccos \frac{1 - x^2}{1 + x^2}$. 3.14. $f(x) = \cos (2 \arccos x)$.

3.15. $f(x) = e^{\sqrt{\frac{1+x}{1-x}}}$. 3.16. $f(x) = e^{\operatorname{tg} \frac{1}{x}}$.

3.17. $f(x) = \operatorname{tg}^2 x + \ln \cos^2 x$. 3.18. $f(x) = \operatorname{arctg} (\operatorname{ctg}^2 x)$.

3.19. $f(x) = \sqrt{2x^2 + \sqrt{x^2 + 1}}$. 3.20. $f(x) = \sqrt{2 + x^2} \sqrt[3]{3 + x^3}$.

4. Să se calculeze derivata funcției:

4.1. $f(x) = \ln(2x - 3 + \sqrt{4x^2 - 12x + 10}) - \operatorname{arctg}(2x - 3)\sqrt{4x^2 - 12x + 10}.$

4.2. $f(x) = x^2\sqrt{x^4 + 1} + \ln(x^2 + \sqrt{x^4 + 1}).$

4.3. $f(x) = x + e^{-x} \operatorname{arctg} e^x - \ln\sqrt{1 + e^{2x}}.$

4.4. $f(x) = \sqrt{49x^2 + 1} \operatorname{arctg} 7x - \ln(7x + \sqrt{49x^2 + 1}).$

4.5. $f(x) = \arcsin e^{-2x} + \ln(e^{2x} + \sqrt{e^{4x} - 1}).$

4.6. $f(x) = \frac{3 - \sin x}{2} \sqrt{\cos^2 x - 2 \sin x} + 2 \arcsin \frac{1 + \sin x}{\sqrt{2}}.$

4.7. $f(x) = \operatorname{arctg} \sqrt{e^x} + e^x \arcsin \sqrt{\frac{e^x}{e^x + 1}} - \sqrt{e^x}.$

4.8. $f(x) = 2\sqrt{3} \operatorname{arctg} \frac{\sqrt{3}}{1 - 2x^2} + \ln \frac{x^4 - x^2 + 1}{x^4 + 2x^2 + 1}.$

4.9. $f(x) = \ln \frac{2(x^2 + 2x + 2)}{2x^2 + 2x + 1} + 4 \operatorname{arctg}(x + 1) - \operatorname{arctg}(2x + 1).$

4.10. $f(x) = \frac{5x + 2}{x^2 + x + 1} + \ln \sqrt[3]{\frac{(x - 1)^2}{x^2 + x + 1}} + \frac{8}{\sqrt{3}} \operatorname{arctg} \frac{2x + 1}{\sqrt{3}}.$

4.11. $f(x) = x \ln(\sqrt{1 - x} + \sqrt{1 + x}) + \frac{1}{2}(\arcsin x - x).$

4.12. $f(x) = (3x - 2)^4 \arcsin \frac{1}{3x - 2} + (3x^2 - 4x + 2) \sqrt{9x^2 - 12x + 3}.$

4.13. $f(x) = e^{2 \arcsin x} [\cos(2 \arcsin x) + \sin(2 \arcsin x)].$

4.14. $f(x) = \sqrt{1 + \sqrt[3]{1 + \sqrt[4]{1 + x^4}}}.$

4.15. $f(x) = \frac{2}{3x - 2} \sqrt{12x - 9x^2 - 3} + \ln \frac{1 + \sqrt{12x - 9x^2 - 3}}{3x - 2}.$

4.16. $f(x) = x(2x^2 + 5) \sqrt{x^2 + 1} + 3 \ln(x + \sqrt{x^2 + 1}).$

4.17. $f(x) = \sqrt{x^2 + 5x + 4} + 3 \ln(\sqrt{x + 4} + \sqrt{x + 1}).$

$$4.18. \quad f(x) = \frac{x \arcsin 2x}{\sqrt{1-4x^2}} + \ln \sqrt{1-4x^2}.$$

$$4.19. \quad f(x) = \frac{1}{4\sqrt{3}} \ln \frac{\sqrt{x^2+2} - x\sqrt{3}}{\sqrt{x^2+2} + x\sqrt{3}} + \frac{1}{2} \operatorname{arctg} \frac{\sqrt{x^2+2}}{x}.$$

$$4.20. \quad f(x) = \frac{\cos x}{3(2+\sin x)} + \frac{4}{3\sqrt{3}} \operatorname{arctg} \frac{2 \operatorname{tg} \frac{x}{2} + 1}{\sqrt{3}}.$$

$$4.21. \quad f(x) = \frac{1}{\cos x} + \frac{1}{3 \cos^3 x} - \frac{1}{2} \ln \frac{1 + \cos x}{1 - \cos x}.$$

$$4.22. \quad f(x) = 2\sqrt{1-x^2} \arcsin x - 2x + x(\arcsin x)^2.$$

$$4.23. \quad f(x) = \frac{\ln(1+\sin x)}{\operatorname{tg} x} + x - \ln \operatorname{tg} \frac{x}{2}.$$

$$4.24. \quad f(x) = \log_{\frac{1}{2}} \left(x + \frac{1}{2}\right)^2 + \log_2 \sqrt{4x^2 + 4x + 1}.$$

$$4.25. \quad f(x) = x^x.$$

$$4.26. \quad f(x) = \sin x^{\cos x}.$$

$$4.27. \quad f(x) = x + x^x + x^{x^x}.$$

$$4.28. \quad f(x) = x^{e^x}.$$

$$4.29. \quad f(x) = x^{e^{\sin x}}.$$

$$4.30. \quad f(x) = x^{3^x} 2^x.$$

5. Să se studieze derivabilitatea următoarelor funcții:

5.1. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = |x^3 - 4x|.$

5.2. $f : \left(-\frac{1}{3}, \infty\right) \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} \ln(1 + 3x), & \text{dacă } -\frac{1}{3} < x \leq 0 \\ 3x, & \text{dacă } x > 0. \end{cases}$

5.3. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} \sin^3 x \operatorname{sgn} x, & \text{dacă } |x| \leq \frac{\pi}{4} \\ \frac{3\sqrt{2}}{4}x \operatorname{sgn} x - \frac{\sqrt{2}(3\pi - 4)}{4}, & \text{dacă } |x| > \frac{\pi}{4}. \end{cases}$

5.4. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} \operatorname{tg} \left(x^3 + x^2 \sin \frac{2}{x}\right), & x \neq 0 \\ 0, & x = 0. \end{cases}$

5.5. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} \sqrt[3]{1 - 2x^3 \sin \frac{5}{x}} - 1 + x, & x \neq 0 \\ 0, & x = 0. \end{cases}$

5.6. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \frac{|x + 1| - |4 - x|}{|x| + |x - 5|}.$

5.7. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = |\cos x|.$

5.8. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} x, & \text{dacă } x \in \mathbb{Q} \\ 0, & \text{dacă } x \in \mathbb{R} \setminus \mathbb{Q}. \end{cases}$

5.9. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} \operatorname{arctg} ax, & \text{dacă } |x| \leq 1, a \in \mathbb{R} \\ b \operatorname{sgn} x + \frac{x - 1}{2}, & \text{dacă } |x| > 1, b \in \mathbb{R}. \end{cases}$

5.10. $f : \mathbb{R} \longrightarrow \mathbb{R}, \quad f(x) = \begin{cases} 2^{\frac{1}{x-1}}, & \text{dacă } x < 1 \\ 0, & \text{dacă } x = 1 \\ \ln(x^2 - 2x + 2), & \text{dacă } x > 1. \end{cases}$

6. Să se calculeze derivatele de ordinul n ($n \in \mathbb{Z}, n \geq 1$) ale funcțiilor următoare:

- 6.1. $f(x) = xe^{2x}$. 6.2. $f(x) = \frac{12x - 1}{6x - 1}$.
- 6.3. $f(x) = \sin 3x + \cos(x + 2)$. 6.4. $f(x) = \ln(x + 3)$.
- 6.5. $f(x) = (x - 1)^n(x - 2)^n$. 6.6. $f(x) = x^n e^{-x}$.
- 6.7. $f(x) = \sin x$. 6.8. $f(x) = \cos x$.
- 6.9. $f(x) = \sin^2 x$. 6.10. $f(x) = \sin^4 x + \cos^4 x$.
- 6.11. $f(x) = \frac{x}{x^2 - 4x - 12}$. 6.12. $f(x) = \frac{3}{x^2 - x - 2}$.
- 6.13. $f(x) = x \sin x$. 6.14. $f(x) = \operatorname{arctg} x$.
- 6.15. $f(x) = \frac{1}{\sqrt{x - 1}}$. 6.16. $f(x) = e^x \sin x$.
- 6.17. $f(x) = e^x \cos 2x$. 6.18. $f(x) = \frac{\ln x}{x}$.
- 6.19. $f(x) = \frac{2x + 1}{3x + 2}$. 6.20. $f(x) = \sqrt[3]{e^{2x-1}}$.

7. Utilizând diferențiale, să se calculeze cu aproximație:

- 7.1. $f(x) = x^5$, $x = 3,01$. 7.2. $f(x) = x^6$, $x = 1,997$.
- 7.3. $f(x) = \sqrt[3]{x^2}$, $x = 1,029$. 7.4. $f(x) = \sqrt{3 + x + \cos x}$, $x = 0,01$.
- 7.5. $f(x) = \sqrt{\frac{3 - x}{1 + x}}$, $x = -0,85$. 7.6. $f(x) = \frac{1}{\sqrt{3x + 2}}$, $x = 0,668$.
- 7.7. $f(x) = \arcsin x$, $x = 0,08$. 7.8. $f(x) = \operatorname{arctg} x$, $x = 1,03$.
- 7.9. $f(x) = \sin x$, $x = 31^\circ$. 7.10. $f(x) = \ln \operatorname{tg} x$, $x = 48^\circ$.
- 7.11. $f(x) = \frac{x + \sqrt{10 - x^2}}{2}$, $x = 0,99$. 7.12. $f(x) = \sqrt{x^2 + 12}$, $x = 1,98$.

8. Să se calculeze derivata y'_x :

$$8.1. \quad \begin{cases} x = \sin^2 t, \\ y = \cos^2 t. \end{cases}$$

$$8.3. \quad \begin{cases} x = \sqrt{t}, \\ y = \sqrt[3]{t}. \end{cases}$$

$$8.5. \quad \begin{cases} x = \frac{3at}{1+t^3}, \\ y = \frac{3at^2}{1+t^3}. \end{cases}$$

$$8.7. \quad \begin{cases} x = \operatorname{arctg} e^t, \\ y = \sqrt{e^{2t} + 1}. \end{cases}$$

$$8.9. \quad \begin{cases} x = \frac{t}{1-t^2} \arcsin t + \ln \sqrt{1-t^2}, \\ y = \frac{t}{\sqrt{1-t^2}}. \end{cases}$$

$$8.11. \quad \begin{cases} x = \frac{5t^2 + 2}{5t^3}, \\ y = \sin \left(\frac{1}{3}t^3 + t \right). \end{cases}$$

$$8.13. \quad \begin{cases} x = e^t \cos t, \\ y = e^t \sin t. \end{cases}$$

$$8.2. \quad \begin{cases} x = e^{-t}, \\ y = t^2. \end{cases}$$

$$8.4. \quad \begin{cases} x = e^t, \\ y = \arcsin t. \end{cases}$$

$$8.6. \quad \begin{cases} x = \frac{1}{t+1}, \\ y = \frac{t}{t+1}. \end{cases}$$

$$8.8. \quad \begin{cases} x = \operatorname{arctg} t, \\ y = \ln \frac{1+t^2}{t+1}. \end{cases}$$

$$8.10. \quad \begin{cases} x = \ln \operatorname{tg} t, \\ y = \operatorname{cosec}^2 t. \end{cases}$$

$$8.12. \quad \begin{cases} x = \sqrt{4-t^2}, \\ y = \operatorname{tg} \sqrt{2+t}. \end{cases}$$

$$8.14. \quad \begin{cases} x = a(\sin t - t \cos t), \\ y = a(\cos t + t \sin t). \end{cases}$$

8.15. $xy + \ln y = 1.$

8.16. $\sqrt{x} + \sqrt{y} = 1.$

8.17. $\frac{x^2}{9} + \frac{y^2}{4} = 1.$

8.18. $e^y + xy = 2e.$

8.19. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}.$

8.20. $y^5 + y^3 + y - x = 0.$

8.21. $\operatorname{arctg} \frac{y}{x} = \ln \sqrt{x^2 + y^2}.$

8.22. $y^2 = 2px.$

8.23. $x^2 + y^2 - 6x + 10y - 2 = 0.$

8.24. $x^2y + \operatorname{arctg} \frac{y}{x} = 0.$

9. Să se scrie ecuațiile tangentelor la graficele funcțiilor în punctele specificate:

9.1. $f(x) = x^2 - x - 12, x = 3.$

9.2. $f(x) = \frac{1}{3}(3x - x^3), x = 2.$

9.3. $f(x) = \frac{x^3 + 1}{x^3 - 1}, x = 0.$

9.4. $f(x) = \frac{x}{x^2 + 1}, x = -1.$

9.5. $f(x) = \frac{x^2 - x - 2}{x^2 - 3x}, x = 2.$

9.6. $f(x) = \ln \frac{x^2 - 2x + 1}{x^2 + x + e}, x = 0.$

9.7. $f(x) = \cos 2x - 2 \sin x, x = \frac{\pi}{2}.$

9.8. $f(x) = \operatorname{arctg} \frac{1}{x}, x = 1.$

9.9. $f(x) = \frac{x}{\sqrt[3]{x+1}}, x = -2.$

9.10. $f(x) = 4 \operatorname{tg} x - \frac{\sin x}{\cos^2 x}, x = \frac{\pi}{4}.$

10. Să se determine în ce puncte și sub ce unghi se intersectează graficele funcțiilor:

- 10.1. $f_1(x) = \sin x$, $f_2(x) = \sqrt{3} \cos x$. 10.2. $f_1(x) = x^2$, $f_2(x) = x$.
 10.3. $f_1(x) = x^3$, $f_2(x) = x^2$. 10.4. $f_1(x) = (x-2)^2$, $f_2(x) = 4 - x^2$.
 10.5. $f_1(x) = \frac{1}{\sqrt[3]{x}}$, $f_2(x) = x$. 10.6. $f_1(x) = \frac{1}{x^3}$, $f_2(x) = x^2$.
 10.7. $f_1(x) = 4x^2 + 2x - 8$,
 $f_2(x) = x^3 - x + 10$. 10.8. $f_1(x) = \ln x$, $f_2(x) = 2 - \frac{x}{e}$.
 10.9. $f_1(x) = 3x - x^2$, $f_2(x) = x^2 - x$. 10.10. $f_1(x) = \sin x$, $f_2(x) = \cos x$.

11. Să se studieze monotonia și să se determine punctele de extrem pentru fiecare din funcțiile f pe domeniul lor maxim de definiție:

- 11.1. $f(x) = x^2 - x - 12$. 11.2. $f(x) = 6x - x^2$.
 11.3. $f(x) = 3x^3 - 4x^2 + 1$. 11.4. $f(x) = x^3 - 6x^2 + 2$.
 11.5. $f(x) = (x+1)^2(x-4)^3$. 11.6. $f(x) = x^2 - 8 \ln x$.
 11.7. $f(x) = \sqrt[3]{(2-x)(1-x)^2}$. 11.8. $f(x) = (x-1)\sqrt{x^2-1}$.
 11.9. $f(x) = \ln(1+x) - x + \frac{x^2}{2}$. 11.10. $f(x) = \frac{x^2}{x-1}$.
 11.11. $f(x) = \ln \sqrt{1+x^2} + \operatorname{arctg} x$. 11.12. $f(x) = x^2 e^{\frac{1}{x}}$.
 11.13. $f(x) = \ln(4x^2+1) - 8 \operatorname{arctg} 2x$. 11.14. $f(x) = \frac{x^3}{3} e^{-x}$.
 11.15. $f(x) = \ln x + \operatorname{arctg} x$. 10.16. $f(x) = x^2 \ln x$.
 11.17. $f(x) = x - 2 \operatorname{arctg}(x-1) - 1$. 11.18. $f(x) = \sin^3 x + \cos^3 x$.
 11.19. $f(x) = \cos x + \frac{1}{2} \sin 2x$. 11.20. $f(x) = \frac{1}{x-1} + \frac{2(x-1)}{x^2-2x}$.

12. Să se determine intervalele de concavitate, convexitate și eventualele puncte de inflexiune pentru funcțiile următoare:

- 12.1. $f(x) = 2x^4 - 3x^2 + 3x - 2.$ 12.2. $f(x) = x^4 + 4x^3.$
12.3. $f(x) = 3x^2 - x^3 + 1.$ 12.4. $f(x) = x + \cos x.$
12.5. $f(x) = e^{-x^2} + 2x.$ 12.6. $f(x) = \ln(1 + x^2).$
12.7. $f(x) = \frac{(x+1)^2}{x^3}.$ 12.8. $f(x) = \frac{\ln(x+1)}{\sqrt{x+1}}.$
12.9. $f(x) = \left(\frac{x}{2-x}\right)^4.$ 12.10. $f(x) = \sin x + \frac{1}{3}\sin 3x.$
12.11. $f(x) = \sqrt[3]{x-1} - \sqrt[3]{x}.$ 12.12. $f(x) = \sin x - \sin^3 x.$
12.13. $f(x) = \sin^4 x - \cos^4 x.$ 12.14. $f(x) = x^5 - 10x^2 + 7x.$
12.15. $f(x) = \operatorname{tg} x + \cos x.$ 12.16. $f(x) = x + \ln x^2.$
12.17. $f(x) = \ln \frac{x}{x-3}.$ 12.18. $f(x) = \frac{\sqrt{x+1}}{x}.$
12.19. $f(x) = e^x - \frac{1}{2}x^2 + 1.$ 12.20. $f(x) = 3x + 2\sin \frac{x}{2}.$

13. Să se reprezinte grafic următoarele funcții, $f : D \rightarrow \mathbb{R}$, D – fiind domeniul maxim de definiție:

- 13.1. $f(x) = 3x - x^3.$ 13.2. $f(x) = 2 - 3x - x^3.$
13.3. $f(x) = \frac{1}{16}x^2(x-4)^2.$ 13.4. $f(x) = x^2 - x^4.$
13.5. $f(x) = x(2x^2 + 9x + 12).$ 13.6. $f(x) = (x-1)^2(3-x)^2.$
13.7. $f(x) = \frac{3x-2}{x^3}.$ 13.8. $f(x) = \frac{x^3+4}{x^2}.$

$$\begin{array}{ll}
13.9. & f(x) = \left(\frac{x}{x-1}\right)^2. \\
13.10. & f(x) = \frac{3x^4 + 1}{x^3}. \\
13.11. & f(x) = \frac{x^3}{x-1}. \\
13.12. & f(x) = 3x + \frac{6}{x} - \frac{1}{x^3}. \\
13.13. & f(x) = \frac{3}{x+2} - \frac{3}{x-2} - 1. \\
13.14. & f(x) = \frac{\ln(x+1)}{\sqrt{x+1}}. \\
13.15. & f(x) = \sin x - \sin^2 x. \\
13.16. & f(x) = \cos 3x + 3 \cos x. \\
13.17. & f(x) = \sin x + \frac{1}{2} \sin 2x. \\
13.18. & f(x) = \cos x \cos 3x. \\
13.19. & f(x) = \arccos \frac{2x}{1+x^2}. \\
13.20. & f(x) = \arcsin \frac{1-x^2}{1+x^2}. \\
13.21. & f(x) = \ln x - x + 1. \\
13.22. & f(x) = x^2 \ln x. \\
13.23. & f(x) = \frac{\ln x}{x}. \\
13.24. & f(x) = \ln \left(\frac{x-5}{x}\right) + 2. \\
13.25. & f(x) = x \operatorname{arctg} x. \\
13.26. & f(x) = \operatorname{arctg} \sin x. \\
13.27. & f(x) = \ln(\sin x - \cos x). \\
13.28. & f(x) = x^{\frac{2}{3}} e^{-\frac{x^2}{3}}. \\
13.29. & f(x) = \frac{e^{x+2}}{x+2}. \\
13.30. & f(x) = e^{\sin x + \cos x}. \\
13.31. & f(x) = \sqrt[3]{x(x^2-1)}. \\
13.32. & f(x) = \sqrt[3]{(x-2)(x+1)^2}.
\end{array}$$

14. Să se calculeze limitele următoare folosind regula lui l'Hospital:

$$\begin{array}{ll}
14.1. & \lim_{x \rightarrow 1} \frac{x^3 - 5x^2 + 4}{2x^3 - x^2 - 1}. \\
14.2. & \lim_{x \rightarrow 1} \frac{x^5 - 1}{\ln x}. \\
14.3. & \lim_{x \rightarrow 0} \frac{\sin 5x}{2x}. \\
14.4. & \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 5\pi x}{\cos 3\pi x}. \\
14.5. & \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - x}{\sin x - x}. \\
14.6. & \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{\ln(1+x)}.
\end{array}$$

- 14.7. $\lim_{x \rightarrow \infty} \frac{\ln(1 + \frac{1}{x^2})}{\pi - 2 \operatorname{arctg} x}$.
- 14.8. $\lim_{x \rightarrow 0} \frac{\ln \cos 2x}{\ln \cos 3x}$.
- 14.9. $\lim_{x \rightarrow 0} \frac{\sin 2x - 2xe^x + 3x^2}{\operatorname{arctg} x - \sin x - \frac{x^3}{6}}$.
- 14.10. $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\ln \operatorname{tg} x}{\operatorname{ctg} 2x}$.
- 14.11. $\lim_{x \rightarrow \infty} \frac{\pi - 2 \operatorname{arctg} x}{e^{\frac{2}{x}} - 1}$.
- 14.12. $\lim_{x \rightarrow 1} \frac{\ln x - x + 1}{\operatorname{tg}^2(x - 1)}$.
- 14.13. $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$.
- 14.14. $\lim_{x \rightarrow \infty} \frac{x^4}{e^x}$.
- 14.15. $\lim_{x \rightarrow 1+} \frac{\ln(x - 1)}{\operatorname{ctg} \pi x}$.
- 14.16. $\lim_{x \rightarrow 0+} \frac{\ln x}{\ln \sin x}$.
- 14.17. $\lim_{x \rightarrow 0} x \operatorname{ctg} \pi x$.
- 14.18. $\lim_{x \rightarrow \frac{\pi}{2}} \left(x - \frac{\pi}{2}\right) \operatorname{tg} x$.
- 14.19. $\lim_{x \rightarrow 0} (\operatorname{ctg} x \arcsin x)$.
- 14.20. $\lim_{x \rightarrow 0} \sin x \ln(\operatorname{ctg} x)$.
- 14.21. $\lim_{x \rightarrow 2} (x - 2) \operatorname{tg} \frac{\pi x}{4}$.
- 14.22. $\lim_{x \rightarrow 3} (x - 3) \operatorname{ctg} \frac{\pi x}{3}$.
- 14.23. $\lim_{x \rightarrow 1} \left(\frac{1}{x - 1} - \frac{1}{\ln x}\right)$.
- 14.24. $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \operatorname{ctg}^2 x\right)$.
- 14.25. $\lim_{x \rightarrow 3} \left(\frac{2x - 3}{x^2 - 7x + 12} - \frac{1}{(x - 2) \ln(x - 2)}\right)$.
- 14.26. $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\arcsin x}\right)$.
- 14.27. $\lim_{x \rightarrow 1} \left(\frac{2}{1 - x^2} - \frac{3}{1 - x^3}\right)$.
- 14.28. $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^{2x} - 1}\right)$.
- 14.29. $\lim_{x \rightarrow 0} (\cos x)^{\frac{1}{x^2}}$.
- 14.30. $\lim_{x \rightarrow \infty} \left(\frac{2}{\pi} \operatorname{arctg} x\right)^x$.
- 14.31. $\lim_{x \rightarrow 0} (x + 3^x)^{\frac{2}{x}}$.
- 14.32. $\lim_{x \rightarrow 0} (x + e^x)^{\frac{1}{x}}$.
- 14.33. $\lim_{x \rightarrow \pi+} (x - \pi)^{\sin x}$.
- 14.34. $\lim_{x \rightarrow 0+} |\ln x|^{\frac{x}{2}}$.
- 14.35. $\lim_{x \rightarrow 0+} \left(\frac{1}{x}\right)^{\sin x}$.
- 14.36. $\lim_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}}$.