Diagrammatic Reasoning

From Peirce's Existential Graphs to AI

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Visualization and Creativity

Paul Halmos, Former president of the AMS:

"Mathematics — this may surprise or shock some — is never deductive in its creation. The mathematician at work makes vague guesses, visualizes broad generalizations, and jumps to unwarranted conclusions. He arranges and rearranges his ideas, and becomes convinced of their truth long before he can write down a logical proof... the deductive stage, writing the results down, and writing its rigorous proof are relatively trivial once the real insight arrives; it is more the draftsman's work not the architect's." *

Albert Einstein, physicist:

"The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be *voluntarily* reproduced and combined... The above-mentioned elements are, in my case, of visual and some of muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage, when the mentioned associative play is sufficiently established and can be reproduced at will." **

*** Halmos (1968). ** Quoted by Hadamard (1945).**

Using Diagrams in Teaching

Observation by Jon Barwise and John Etchemendy about teaching logic at Stanford University: *

In our classes... students would make egregious errors in translating between sentences of English and sentences of first-order logic, errors that would have been inconceivable had they really understood the meanings of both sentences.

We were reminded, too, that over the years a handful of logicians, most notably Euler, Venn, and Peirce, had stressed the importance and interest of nonsentential inference. The diagrams of Euler and Venn, both of which use circles to represent collections of objects, are still widely known and used, even though their expressive power is sorely limited. C. S. Peirce, inspired by the utility of molecular diagrams in reasoning about chemical compounds, developed a more intricate and powerful diagrammatic formalism. While Peirce's system has not won over many human users, it has become an important tool in computer science.

* J. Barwise & J. Etchemendy, Computers, visualization, and the nature of reasoning, http://kryten.mm.rpi.edu/COURSES/LOGAIS02/bar.etch.reasoning.pdf

Discovery Precedes Demonstration

Quotations by George Polya (1954):

- *Demonstrative reasoning is safe, beyond controversy, and final.*
- *Plausible reasoning is hazardous, controversial, and provisional.*
- *Demonstrative reasoning* [*however*] *is incapable of yielding new knowledge about the world.*

Some mathematicians quoted by Polya:

- **Euler:** *The properties of the numbers known today have been mostly discovered by observations... long before their truth has been confirmed by rigid demonstrations.*
- **Laplace:** *Even in the mathematical sciences, our principal instruments to discover the truth are induction and analogy.*

Charles Sanders Peirce (Address to the AMS in 1894, NEM 1 xviii):

Intellectual powers essential to the mathematician: *"Concentration, imagination, and generalization... Did I hear someone say demonstration? Why, my friends, demonstration is but the pavement on which the chariot of the mathematician rolls."*

The Role of Perception in Reasoning

Observation about Euclid's *Elements* by C. S. Peirce:

- Every theorem has a new diagram.
- Every corollary uses the same diagram as the main theorem.

The creative insight is the visualization of a new diagram:

- The statement of Proposition 1 mentioned only a line and a triangle.
- It did not mention circles, their relationships to each other, or their relationships to the line and the triangle.

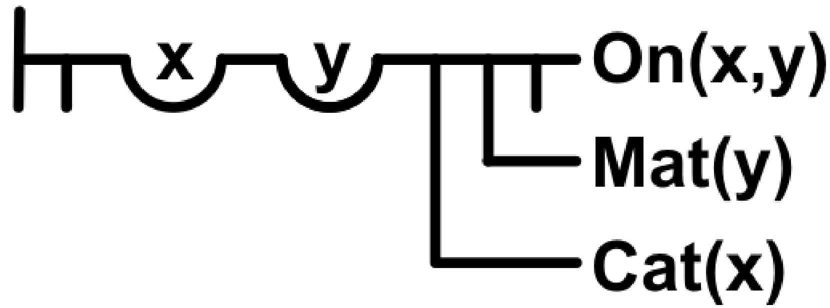
New insights, good or bad, can come from anywhere:

- Taking a bath, walking in the park, or wild images in a dream.
- The diagram is a hypothesis — an abduction that introduces novel ideas. They may be brilliant, irrelevant, or wrong.
- Deduction is systematic perception — a careful examination of the diagram to make implicit relationships explicit.

Peirce's existential graphs (EGs) emphasize the visual insights.

How to say “A cat is on a mat.”

Gottlob Frege (1879):

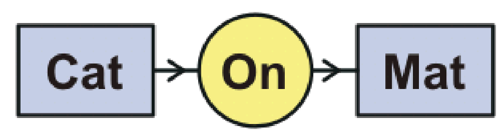


Charles Sanders Peirce (1885): $\Sigma_x\, \Sigma_y\, \mathrm{Cat}_x \bullet \mathrm{On}_{x,y} \bullet \mathrm{Mat}_y$

Giuseppe Peano (1895): $\exists x\, \exists y\, \mathrm{Cat}(x) \wedge \mathrm{On}(x, y) \wedge \mathrm{Mat}(y)$

Existential graph by Peirce (1897): **Cat —— On —— Mat**

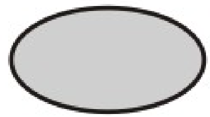
Conceptual graph (1976):



CLIP dialect of Common Logic: **(∃ x y) (Cat x) (On x y) (Mat y).**

Existential Graph Notation (1911)

Existence: —

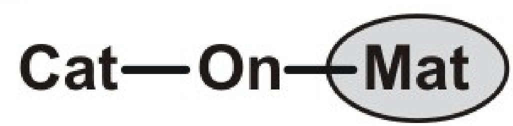
Negation: 

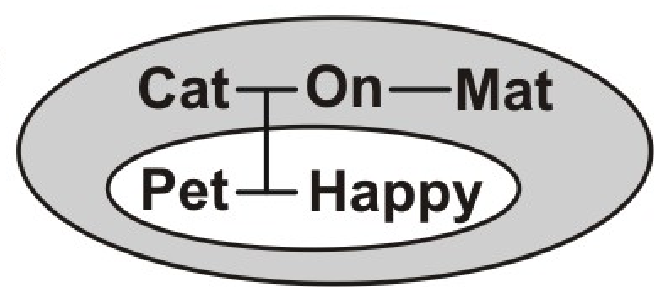
Relations: Cat• Mat• Happy• •On• •Under• •Give•

A cat is on a mat: Cat—On—Mat

Something is under a mat: —Under—Mat

Some cat is not on a mat: 

Some cat is on something that is not a mat: 

If a cat is on a mat, then it is a happy pet: 

Common Logic Interface to Predicate calculus (CLIP)

Existence: (∃ x) or (Exists x)

Negation: ~[ ] but ~[~[ ]] may be written [If [Then]]

Relations: (Cat x), (Mat x), (Pet x), (Happy x), (On x y), (Under x y)

A cat is on a mat: (∃ x y) (Cat x) (On x y) (Mat y).

Something is under a mat: (∃ x y) (Under x y) (Mat y).

Some cat is not on a mat: (∃ x) (Cat x) ~[(∃ y) (On x y) (Mat y)].

Some cat is on something that is not a mat:
(∃ x y) (Cat x) (On x y) ~[(Mat y)].

If a cat is on a mat, then it is a happy pet:
[If (∃ x y) (Cat x) (On x y) (Mat y)
[Then (Pet x) (Happy x)]].

EGs Without Negation

There is a man. *There is a king.* *There is a man who is a king.*

These examples represent existence and relations:

- A *line of identity* states that something exists. In CLIP, that is (∃x).
- Relations in CLIP are represented as (king x), (on x y), (under x y).

Translating the above EGs to CLIP:

(∃x) (man x). *There is something, which is a man*.

(∃x) (king x). *There is something, which is a king*.

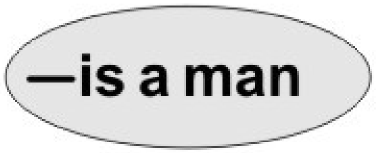
(∃x) (man x) (king x). *There is something, which is a man and a king*.

An option that uses a relation name to *restrict* the quantifier:

(∃ x:man) (king x). *Some man is a king*.

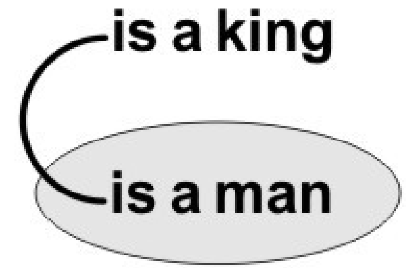
Peirce drew the six EGs in this slide and the next in the manuscript R145, page 21.

EGs With Negation



There is no man.

Something is not a man.



There is a king who is not a man.

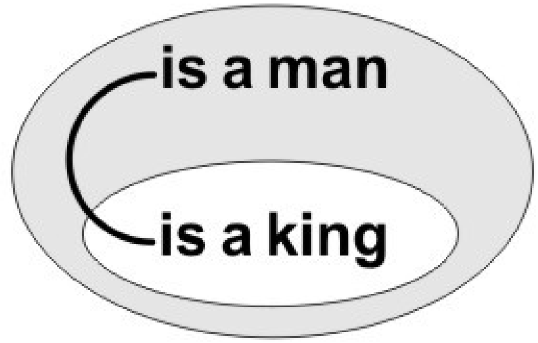
A shaded oval states that the nested graph or subgraph is false:

- **When a line of identity is extended into an oval enclosure, existence is declared in the area that contains the outermost point of the line.**
- **In CLIP, a shaded oval is represented by a tilde ~ and square brackets [ ].**

Translations to CLIP (with two options for the EG on the right):

~[(∃x) (man x)].	*It's false that there is a man*.
(∃x) ~(man x).	*There is something which is not a man*.
(∃x) (king x) ~(man x).	*There is something which is a king and not a man*.
(∃x x:king) ~(man x).	*Some king is not a man*.

Nested Ovals



There is no man who is not a king. *If there is a man, then he is a king.* *Every man is a king.*

An oval nested in an even number of negations is unshaded.

- **Since a double negation is positive, evenly nested areas are positive.**
- **A nest of two ovals represents an if-then statement.**

With two or more negations, an EG may be translated to CLIP or to English in several equivalent ways:

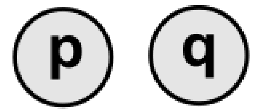
~[(∃x) (man x) ~(king x)].	***It's false that there is a man who is not a king.***
[If (∃ x:man) [Then (king x)]].	***If there is a man, then he is a king*.**
(∀ x:man) (king x).	***Every man x is a king*.**

Boolean Combinations

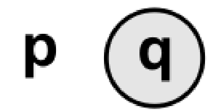
Areas nested inside an odd number of negations are shaded.

p q

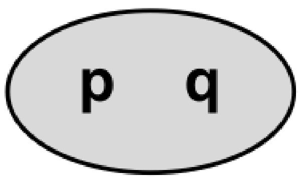
p and q



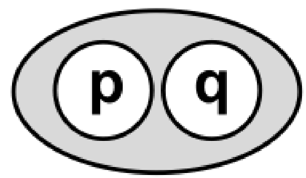
not p and not q



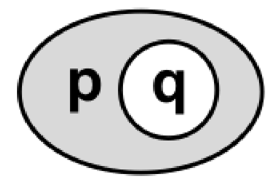
p and not q



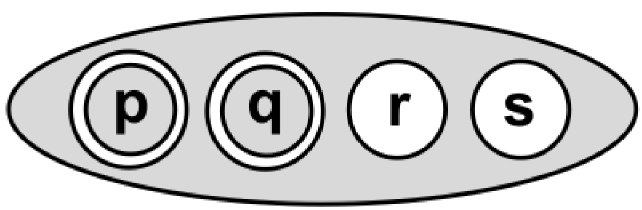
not (p and q)



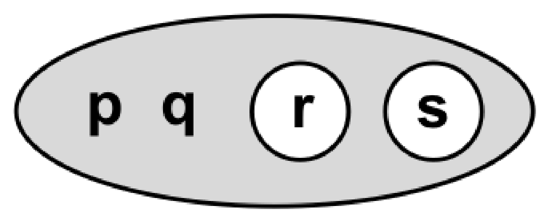
p or q



if p, then q



not p or not q or r or s

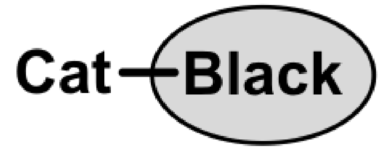


if p and q, then r or s

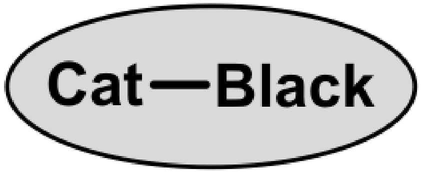
The Scope of Quantifiers

The scope is determined by the outermost point of any line.

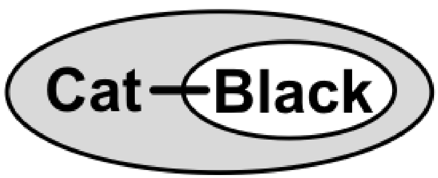
Some cat is black.



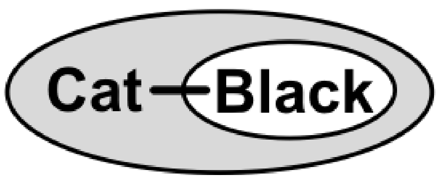
Some cat is not black.



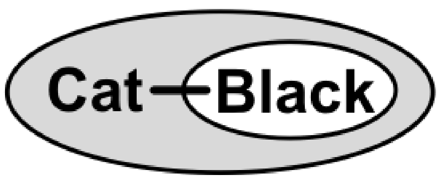
No cat is black.



It is false that some cat is not black.



If there is a cat, then it is black.



Every cat is black.

EGs in these four patterns represent the four sentence types in Aristotle's syllogisms: http://www.jfsowa.com/talks/aristo.pdf

Epistemology

All knowledge is derived from observation.

- Feelings provide information about inner states of the body.
- Sensory organs provide information about external sources.

Only three logical operators can be observed directly.

- Any feeling or sensation is evidence that something exists.
- Any difference or contrast between an observation A and an observation B is evidence of a negation: A is not B.
- Two different observations are evidence of a conjunction: A and B.

No other logical operators can be observed directly.

- The universal quantifier is derived by induction from observations and an assumption that those observations exhaust all possible cases.
- Implication cannot be observed. "Post hoc, ergo propter hoc" is a classical fallacy.
- A disjunction of two or more alternatives cannot be observed. The possibility of options can only be inferred by some method.

Conclusion: Existence, conjunction, and negation are the only logical operators that may be considered primitive.

Syntax of Existential Graphs

Example: Cat —— On —— Mat

- **Two lines mean** ***There exist something x and something y*****.**
- **Cat and Mat are** ***monadic*** **relations. On is a** ***dyadic*** **relation.**

Five syntactic features:

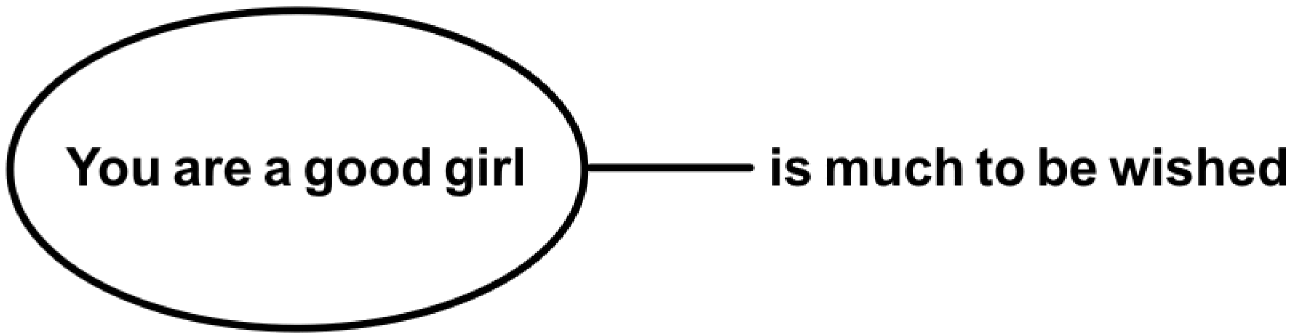
- **Relation: A name with zero or more** ***pegs*** **for attaching lines.**
- **Existence: A** ***line of identity*** **that says** ***Something exists*****.**
- **Conjunction: Two or more graphs in the same area.**
- **Metalanguage: An** ***oval*** **that covers some area.**
- **Negation: A** ***shaded oval*** **that represents the operator** ***not*****.**

Five combinations:

- **Proposition: A graph of lines attached to the pegs of relations.**
- **Identity: Two or more connected lines (called a** ***ligature*****).**
- **Denial: A shaded oval that denies the EG it covers.**
- **Complex Boolean operators: Nests of two or more negations.**
- **Metalanguage: A line that connects an oval to a relation.**

Metalanguage

A relation attached to an oval makes a metalevel comment about the proposition expressed by the nested graph. *



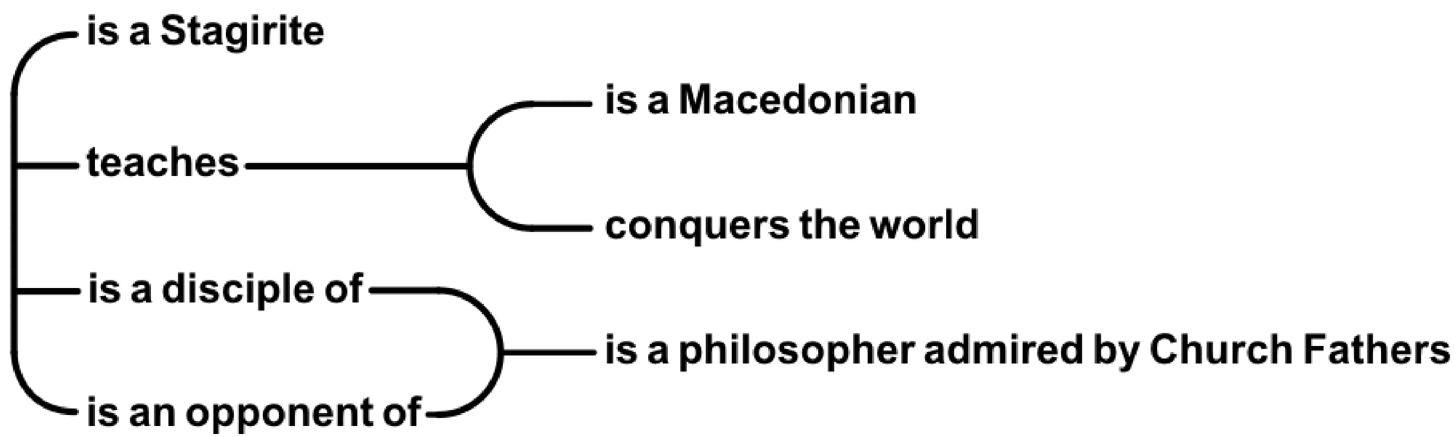
Peirce allowed the names of relations to contain blanks.

The relation named 'You are a good girl' has zero pegs.
It is an EG that expresses a proposition p.

The relation named 'is much to be wished' has one peg, which is attached to a line, which says that the proposition p exists.

* From Charles Sanders Peirce, *Reasoning and the Logic of Things*, The Cambridge Conferences Lectures of 1898, Harvard University Press, p. 151.

One of Peirce's Examples



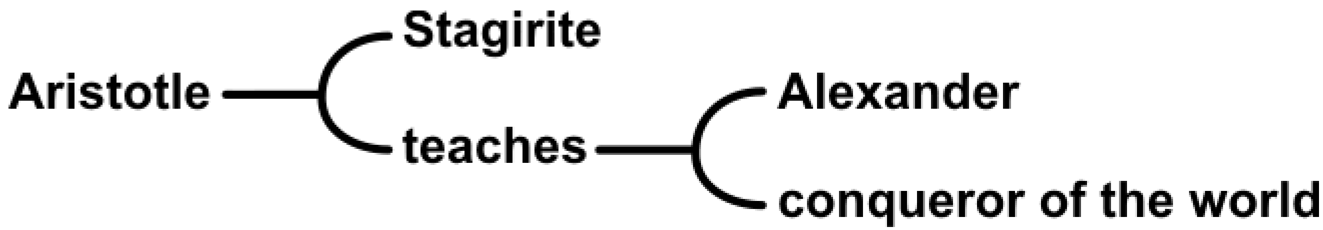
Peirce's translation to English: ***"There is a Stagirite who teaches a Macedonian conqueror of the world and who is at once a disciple and an opponent of a philosopher admired by Fathers of the Church."***

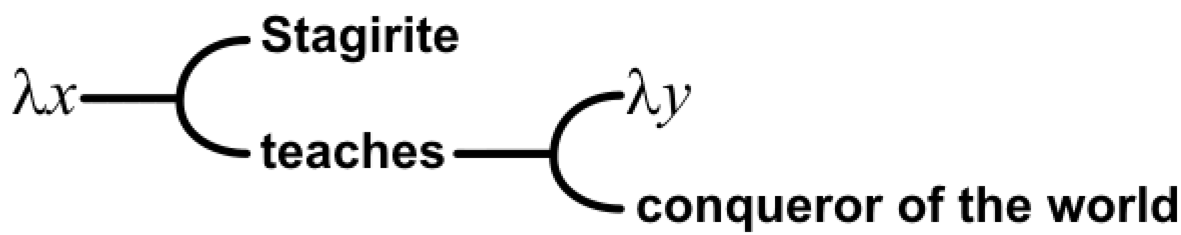
A translation to CLIP:

> **(∃ x y z) ("is a Stagirite" x) (teaches x y) ("is a Macedonian" y) ("conquers the world" y) ("is a disciple of" x z) ("is an opponent of" x z) ("is a philosopher admired by church fathers" z).**

Without negation, CLIP can represent the content of a relational database or the graph databases of the Semantic Web.

Lambda Abstraction





The top EG says ***Aristotle is a Stagirite who teaches Alexander who conquers the world*.**

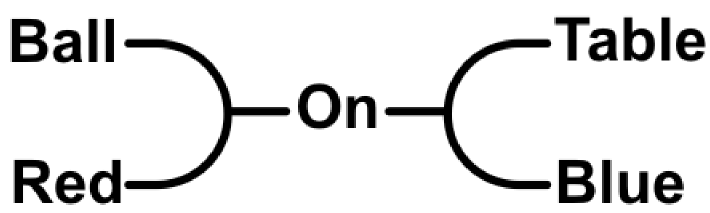
In the EG below it, the names Aristotle and Alexander are erased, and their places are marked with the Greek letter λ.

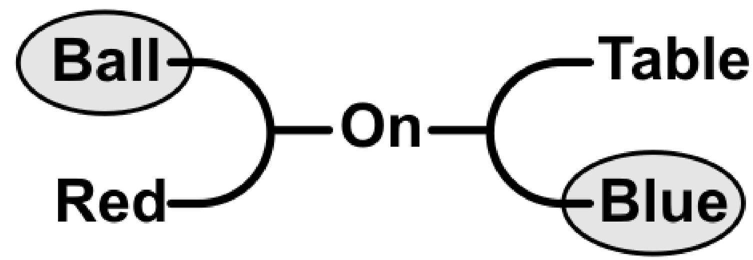
That EG represents a dyadic relation: ___ ***is a Stagirite who teaches*** **___** ***who conquers the world*.**

Peirce used an underscore to mark those empty places, but Alonzo Church marked them with λ.

Translating EGs to and from English

Most existential graphs can be read in several equivalent ways.





Left graph:

A red ball is on a blue table.

Some ball that is red is on some table that is blue.

Right graph:

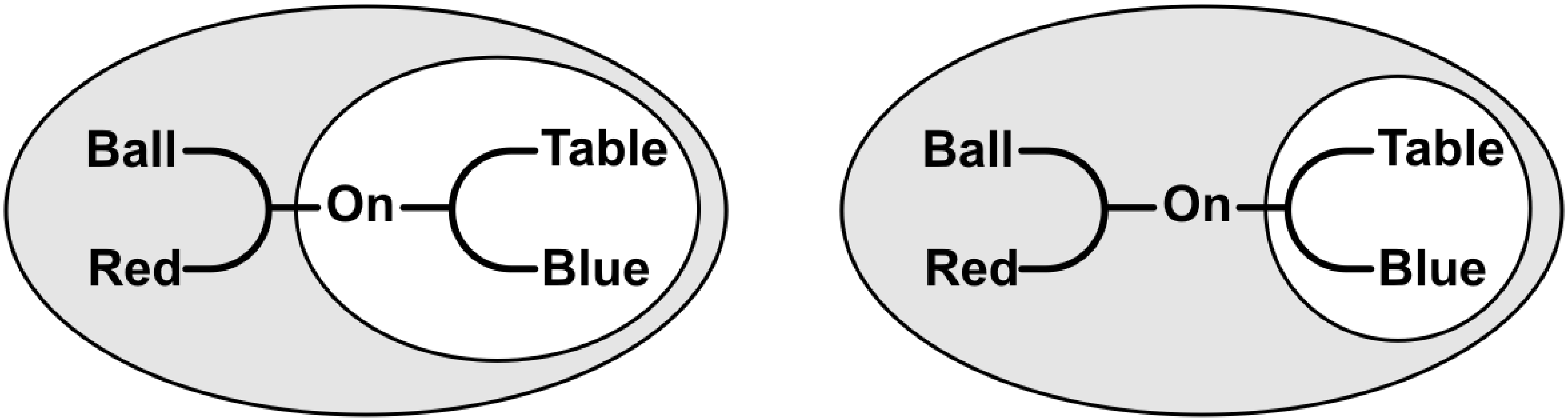
Something red that is not a ball is on a table that is not blue.

A red non-ball is on a non-blue table.

On some non-blue table, there is something red that is not a ball.

Scope of Quantifiers and Negations

Ovals define the scope for both quantifiers and negations.



Left graph:

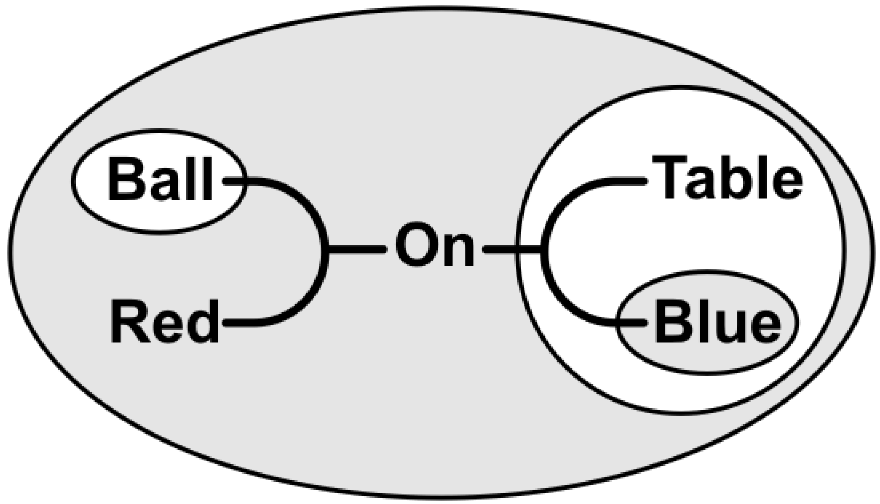
If there is a red ball, then it is on a blue table.

Every red ball is on some blue table.

Right graph:

If a red ball is on something x, then x is a blue table.

EGs With Multiple Nested Negations



The many ways of reading an EG are logically equivalent:

If something red that is not a ball is on something y,
then y is a table that is not blue.

If a red thing x is on something y,
then either x is a ball, or y is a table that is not blue.

If a red thing x is on something that is not a non-blue table,
then x is ball.

Therefore, EGs are a good canonical form for expressing the common meaning. See http://www.jfsowa.com/logic/proposit.pdf

Translating the Word *is* to Logic

Three different translations in English or CLIP:

- **Existence:** *There is x.* ↔ **(∃x).**
- **Predication:** *x is a cat.* ↔ **(Cat x).**
- **Identity:** *x is y.* ↔ **(= x y).**

Do these three translations imply that English is ambiguous?

Or is the syntax of linear notations too complex?

In EGs, all three uses of the word *is* map to a line of identity:

- **Existence:** *There is x.* ↔
- **Predication:** *x is a cat.* ↔ **Cat**
- **Identity:** *x is y.* ↔ **(a ligature of two lines)**

As Peirce said, EGs are more iconic than predicate calculus: they show relationships more clearly and directly.

Issues of Mapping Language to Logic

Hans Kamp observed that predicate calculus (PC) does not have a direct mapping to and from natural languages. *

Pronouns can cross sentence boundaries, but variables cannot.

- **Example:** *Pedro is a farmer. He owns a donkey.*
- **PC: $\exists$x(Pedro(x) $\wedge$ farmer(x)). $\exists$y$\exists$z(owns(y,z) $\wedge$ donkey(z)).**
- **There is no operator that can relate x and y in different formulas.**

In English, quantifiers in the if-clause govern the then-clause.

- **Example:** *If a farmer owns a donkey, then he beats it.*
- **But in predicate calculus, the quantifiers must be moved to the front.**
- **CLIP supports both options: English-like and PC-like.**
  If ($\exists$ x y) (farmer x) (donkey y) (owns x y) [Then (beats x y)]].
  ($\forall$ x y) If (farmer x) (donkey y) (owns x y) [Then (beats x y)]].

Note: Proper names are rarely unique identifiers. Both Kamp and Peirce represented names by monadic predicates.

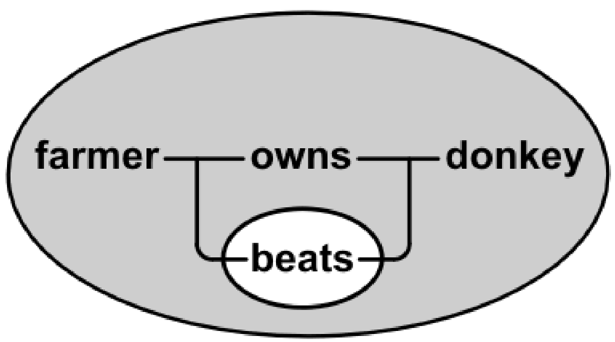
*** Hans Kamp & Uwe Reyle (1993) *From Discourse to Logic*, Dordrecht: Kluwer.**

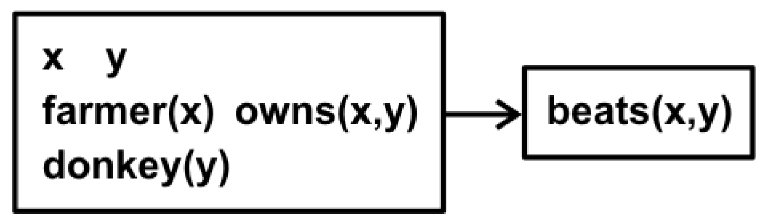
Quantifiers in EG and DRS

Peirce and Kamp independently chose isomorphic structures.

- **Peirce chose nested ovals for EG with lines to show coreference.**
- **Kamp chose boxes for DRS with variables to show coreference.**
- **But the boxes and ovals are isomorphic: they have the same constraints on the scope of quantifiers, and they support equivalent operations.**

Example: ***If a farmer owns a donkey, then he beats it.***





For these examples, CLIP represents the EG and the DRS:
[If (∃ x y) (farmer x) (owns x y) (donkey y) [Then (beats x y)]].

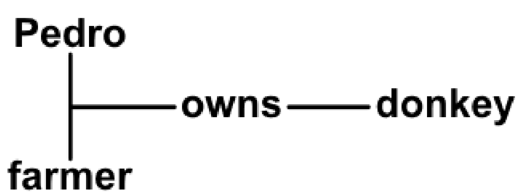
Combining EG Graphs or DRS Boxes

Two English sentences, ***Pedro is a farmer. He owns a donkey,*** **are represented by EG graphs (left) and DRS boxes (right):**

x
Pedro(x) farmer(x)

y z
owns(y,z) donkey(z)

Combine them by connecting EG lines or merging DRS boxes:



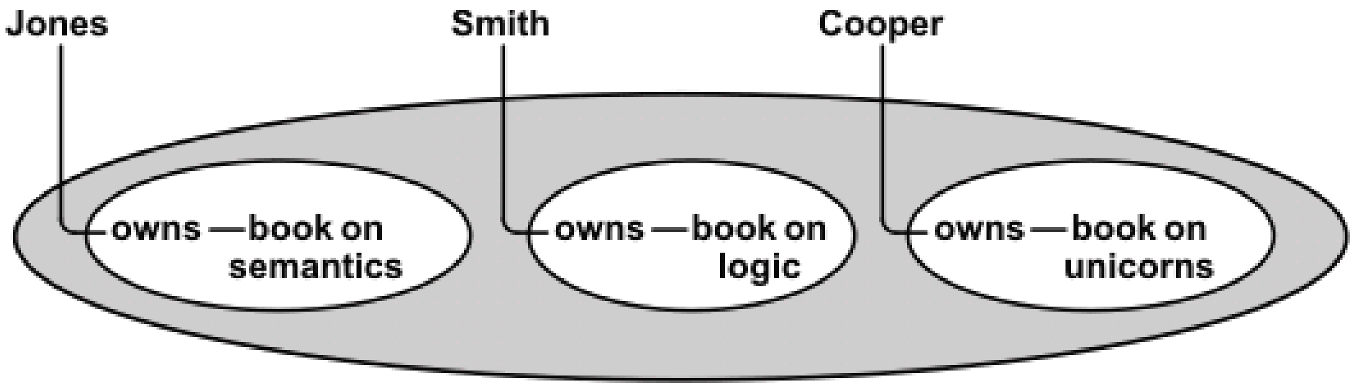
x y z x=y
Pedro(x) farmer(x)
owns(y,z) donkey(z)

Equivalent operations on EG and DRS produce the same CLIP:
(∃ x y z) (Pedro x) (farmer x) (= x y) (owns y z) (donkey z).

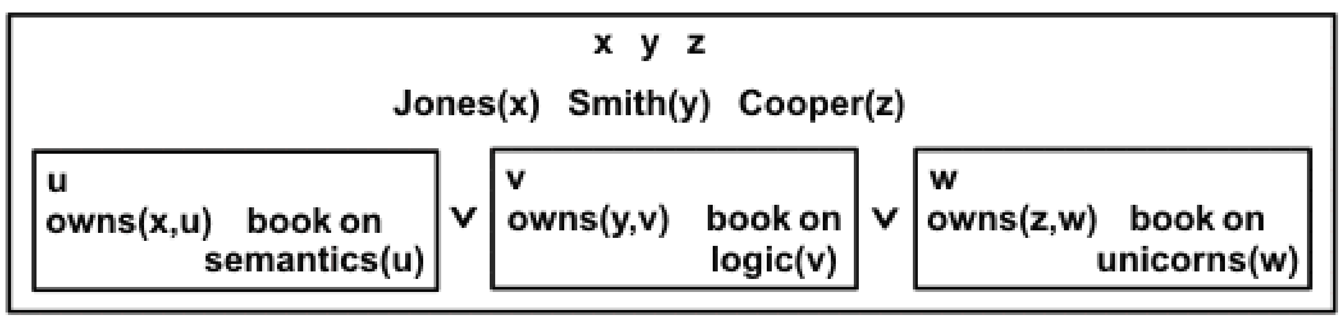
Disjunction in EG, DRS, and CLIP

Kamp and Reyle (1993): ***"Either Jones owns a book on semantics, or Smith owns a book on logic, or Cooper owns a book on unicorns."***

EG:



DRS:



CLIP:

(∃ x y z) (Jones x) (Smith y) (Cooper z)
[Or [(∃ u) (owns x u) ("book on semantics" u)]
[(∃ v) (owns y v) ("book on logic" v)]
[(∃ w) (owns z w) ("book on unicorns" w)]].

Peirce's Rules of Inference

Peirce's rules support the simplest, most general reasoning method ever invented for any logic.

Three pairs of rules, which insert or erase a graph or subgraph:

1. Insert/Erase: Insert anything in a negative area; erase anything in a positive area.
2. Iterate/Deiterate: Iterate (copy) anything in the same area or any nested area; deiterate (erase) any iterated copy.
3. Double negation: Insert or erase a double negation (pair of ovals with nothing between them) around anything in any area.

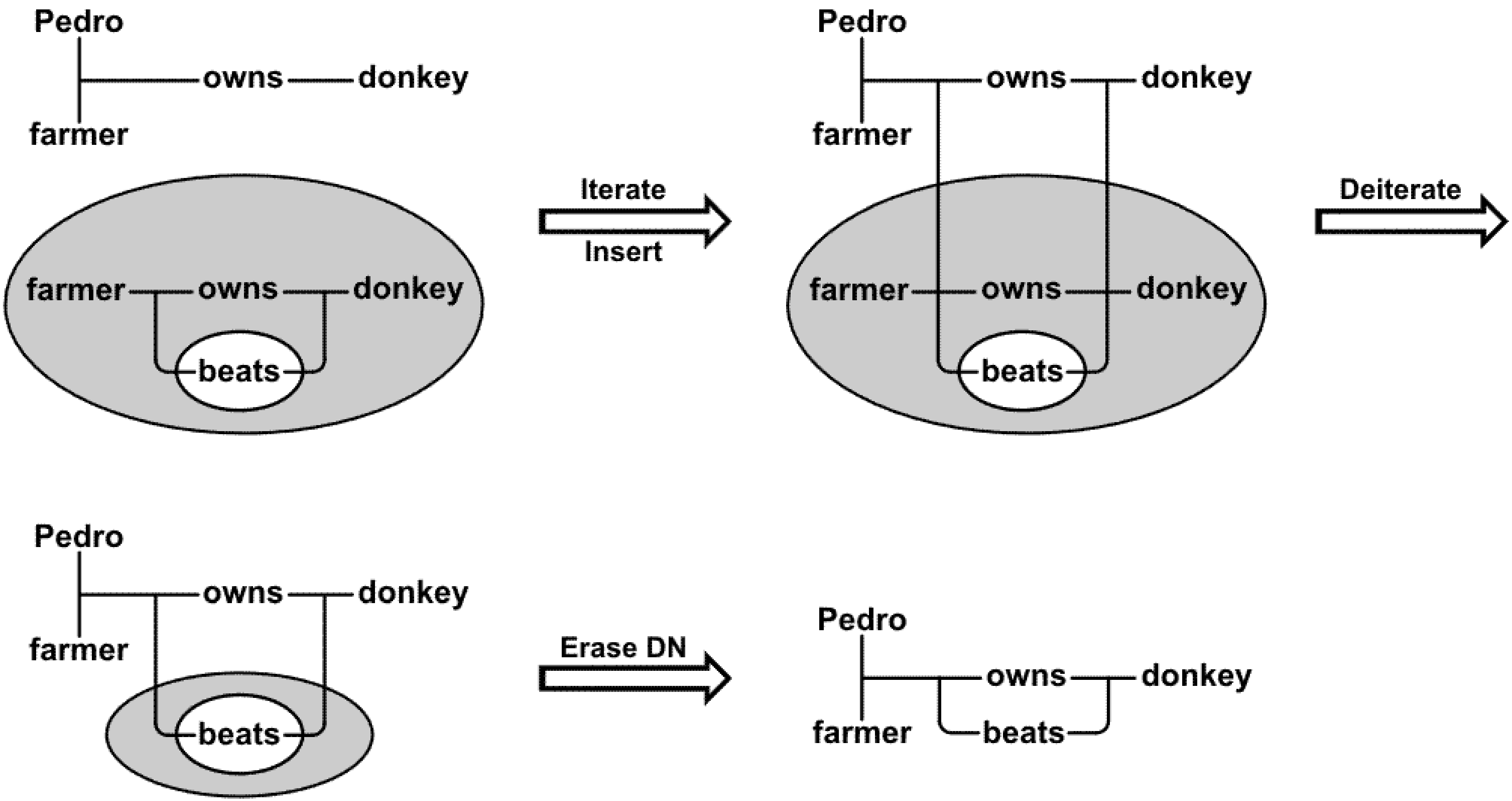
These rules are stated in terms of EGs.

But they can be adapted to many notations, including CLIP, DRS, predicate calculus, various diagrams, and natural languages.

For details, see "Reasoning with diagrams and images"

http://www.collegepublications.co.uk/downloads/ifcolog00025.pdf

A Proof by Peirce's Rules



Conclusion: ***Pedro is a farmer who owns and beats a donkey*.**

Proving a Theorem

Peirce's only axiom is the empty graph – a blank sheet of paper.

- **The empty graph cannot say anything false.**
- **Therefore, the empty graph is always true.**
- **Silence is golden.**

A theorem is a proposition that is proved from the empty graph.

- **For the first step, only one rule can be applied: draw a double negation around a blank area.**
- **The next step is to insert the hypothesis into the negative area.**

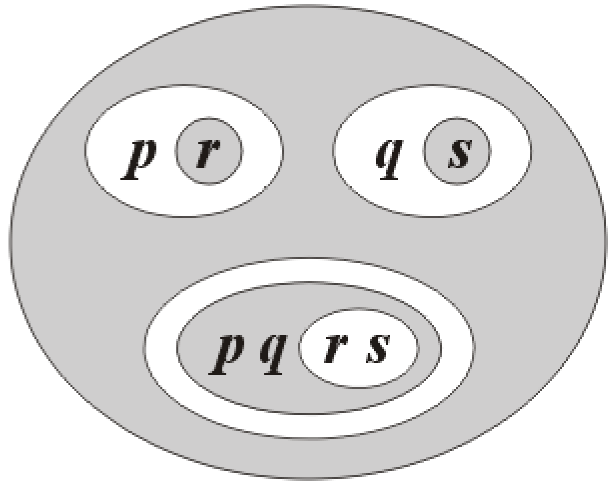
The Praeclarum Theorema (splendid theorem) by Leibniz:

PC: $((p \supset r) \land (q \supset s)) \supset ((p \land q) \supset (r \land s))$.

In the *Principia Mathematica*, Whitehead and Russell took 43 steps to prove this theorem.

With Peirce's rules, the proof takes only 7 steps.

Praeclarum Theorema

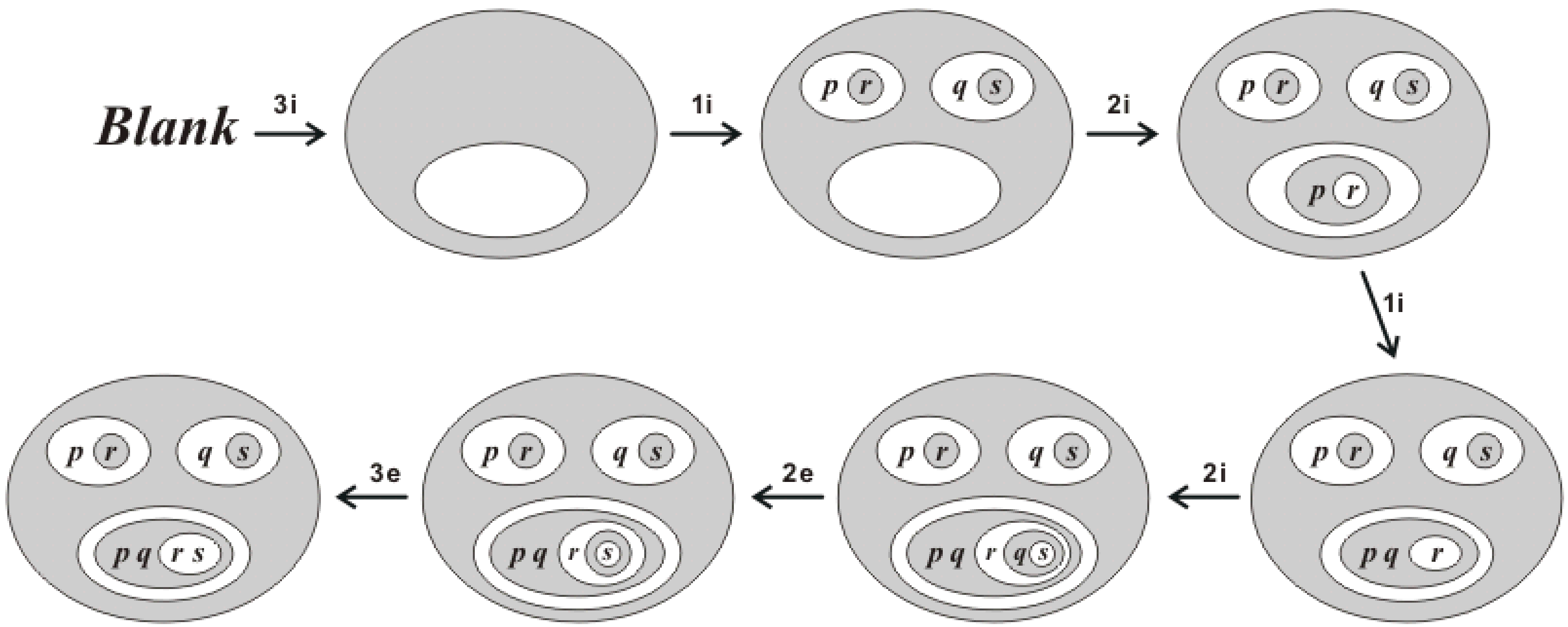


PC: $((p \supset r) \wedge (q \supset s)) \supset ((p \wedge q) \supset (r \wedge s))$

Note that the if-parts of $(p \supset r)$ and $(q \supset s)$ are white, because those areas are nested two levels deep.

But the if-part of $(p \wedge q) \supset (r \wedge s)$ is shaded, because that area is nested three levels deep.

Proof of the Praeclarum Theorema



Each step is labeled with the number of the rule:

> **3i, insert double negation. 1i, insert ((*p*⊃*r*) ∧ (*q*⊃*s*)). 2i, iterate (*p*⊃*r*).**
> **1i, insert *q*. 2i, iterate (*q*⊃*s*). 2e, deiterate q. 3e, erase double negation.**

For humans, perception determines which rule to apply.

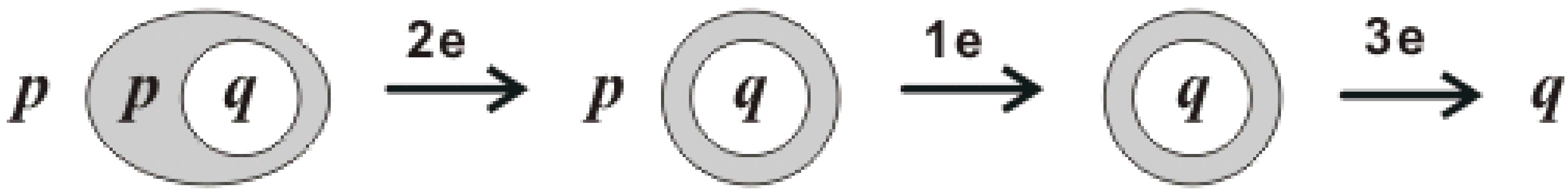
Look ahead to the conclusion to see which rule would make the current graph look more like the target graph.

Proof of the Praeclarum Theorema in CLIP

1. By 3i, draw a double negation around the blank: ~[~[ ]].
2. By 1i, insert the hypothesis in the negative area:
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[ ]].
3. By 2i, iterate the left part of the hypothesis into the conclusion:
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[~[(p) ~[(r)]]]].
4. By 1i, insert (q):
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[~[(p) (q) ~[(r)]]]].
5. By 2i, iterate the right part of the hypothesis into the innermost area:
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[~[(p) (q) ~[(r) ~[(q) ~[(s)]]]]].
6. By 2e, deiterate (q):
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[~[(p) (q) ~[(r) ~[~[(s)]]]]].
7. By 3e, erase the double negation to generate the conclusion:
   ~[~[(p) ~[(r)]] ~[(q) ~[(s)]] ~[~[(p) (q) ~[(r) (s)]]].
8. Replace the negations by the keywords 'If' and 'Then':
   [If [If (p) [Then (r)]] [If (q) [Then (s)]] [Then [If (p) (q) [Then (r) (s)]]].

The CLIP proof is identical to the EG proof, except for readability.

Derived Rules of Inference



Proof of modus ponens: Given *p* and (*p*⊃*q*):

2e, deiterate nested *p*. 1e, erase *p*. 3e, erase double negation.

Therefore, modus ponens may be used as a derived rule of inference in any proof by Peirce's rules.

In general,

- **All rules and proof procedures of classical first-order logic may be derived by a proof that uses Peirce's rules.**
- **Therefore, any or all of those rules may be used as derived rules in any proof that uses EGs.**
- **With appropriate constraints, Peirce's rules may also be adapted to higher-order logics, nonmonotonic logics, intuitionistic logics, etc.**

Applying Peirce's Rules to Other Notations

With minor changes, Peirce's rules can be used with many logic notations, including controlled subsets of natural languages.

Definition: Proposition X is more general (or specialized) than Y iff the models for X are a proper superset (subset) of the models for Y.

Modified version of Peirce's first pair of rules:

- **Insert: In a negative context, any proposition may be replaced by a more specialized proposition.**
- **Erase: In a positive context, any proposition may be replaced by a more general proposition.**

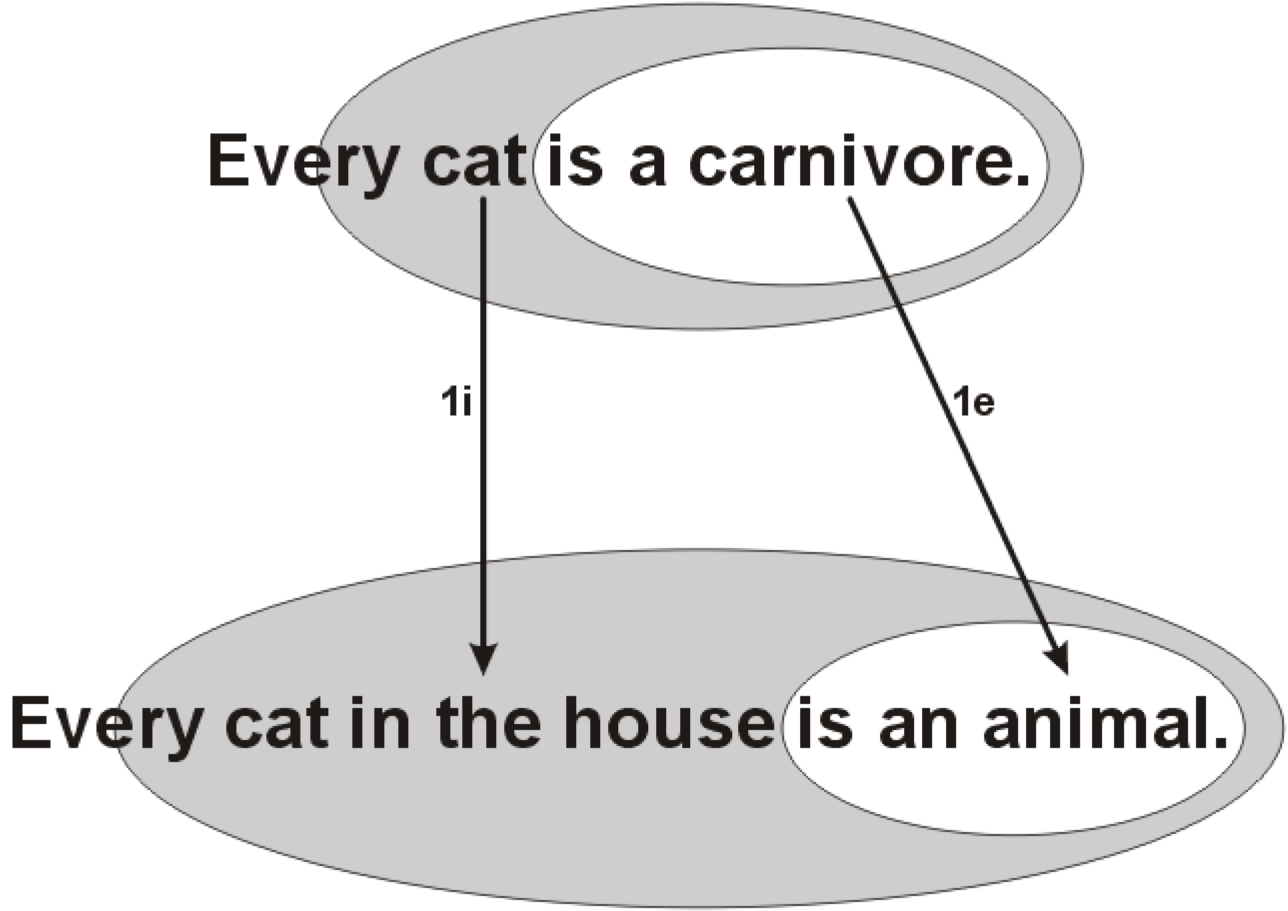
The rules of Iterate/Deiterate and Double Negation are unchanged.

This modification holds for existential graphs, since erasing any subgraph makes a graph more general.

But this version may often be easier to apply to other notations.

Peirce’s Rules Applied to English

Mark the positive and negative parts of each sentence by overlaying shaded ovals.



Rule 1i specializes 'cat' to 'cat in the house'.

Rule 1e generalizes 'carnivore' to 'animal'.

This method of reasoning is sound for sentences that can be mapped to a formal logic. It can also be used on propositional parts of sentences that contain some nonlogical features.

A Proof in English

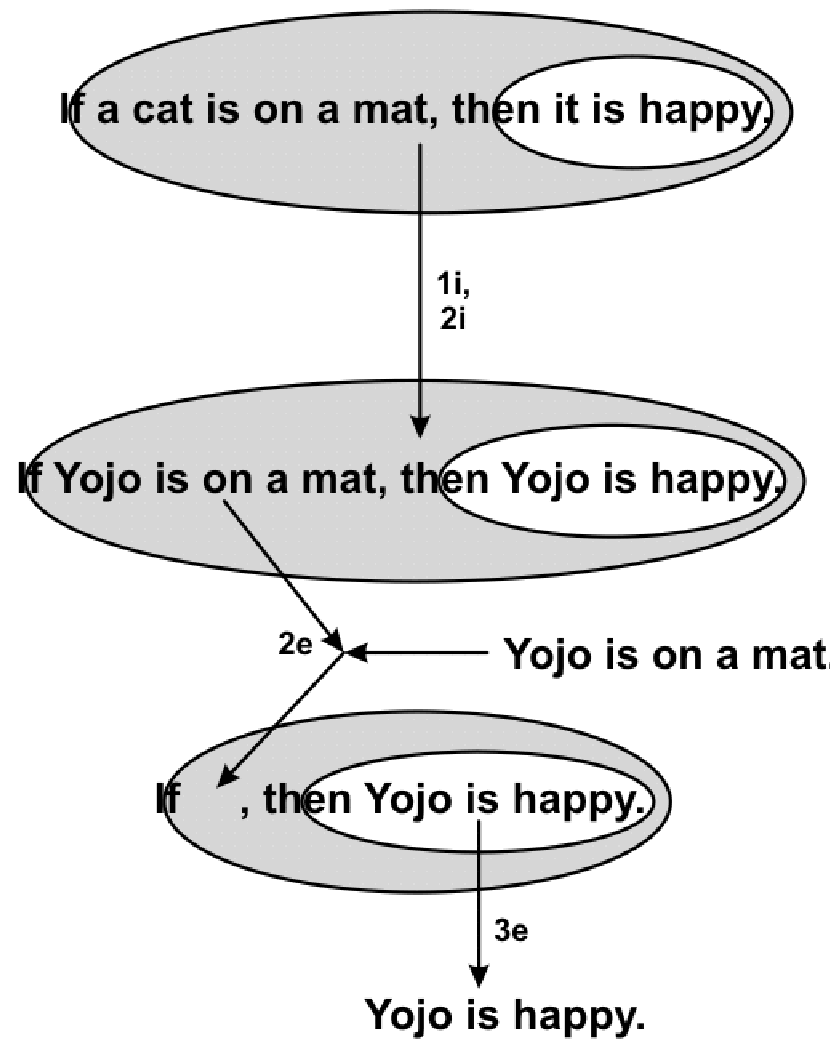
Use shading to mark positive and negative parts of each sentence.

Rule 1i specializes 'a cat' to 'Yojo', and Rule 2i iterates 'Yojo' to replace the pronoun 'it'.

Rule 2e deiterates the nested copy of the sentence 'Yojo is on a mat'.

As a result, there is nothing left between the inner and outer negation of the if-then nest.

Finally, Rule 3e erases the double negation to derive the conclusion.



Natural Deduction

Gerhard Gentzen developed a proof procedure that he called *natural deduction*.

But Peirce’s method is a version of natural deduction that is simpler and more general than Gentzen’s:

Peirce’s Method	Gentzen’s Method
6 rules	16 rules
3 symmetric pairs	Many irregularities
Simple operations	Requires provability
Straight-line proofs	Complex bookkeeping
Date: 1897-1909	Date: 1935

For a proof of equivalence, see http://www.jfsowa.com/pubs/egtut.pdf

Gentzen’s Natural Deduction

	Introduction Rules	Elimination Rules
$\wedge$	$\dfrac{A,\ B}{A \wedge B}$	$\dfrac{A \wedge B}{A}$ $\quad$ $\dfrac{A \wedge B}{B}$
$\vee$	$\dfrac{A}{A \vee B}$ $\quad$ $\dfrac{B}{A \vee B}$	$\dfrac{A \vee B,\ \ A \vdash C,\ \ B \vdash C}{C}$
$\supset$	$\dfrac{A \vdash B}{A \supset B}$	$\dfrac{A,\ \ A \supset B}{B}$
$\sim$	$\dfrac{A \vdash \bot}{\sim A}$ $\quad$ $\dfrac{\bot}{A}$	$\dfrac{A,\ \ \sim A}{\bot}$ $\quad$ $\dfrac{\sim\sim A}{A}$
$\forall$	$\dfrac{A(a)}{(\forall x)A(x)}$	$\dfrac{(\forall x)A(x)}{A(t)}$
$\exists$	$\dfrac{A(t)}{(\exists x)A(x)}$	$\dfrac{(\exists x)A(x),\ \ A(a) \vdash B}{B}$

Like Peirce, Gentzen assumed only one axiom: a blank sheet of paper.

But Gentzen had more operators and more complex pairs of rules for inserting or erasing operators.

Role of the Empty Sheet

Both Peirce and Gentzen start a proof from an empty sheet.

In Gentzen's syntax, a blank sheet is not a well-formed formula.

- Therefore, no rule of inference can be applied to a blank.
- The method of making and discharging an assumption is the only way to begin a proof.

But in EG syntax, an empty graph is a well-formed formula.

- Therefore, a blank may be enclosed in a double negation.
- Then any assumption may be inserted in the negative area.

Applying Peirce's rules to predicate calculus:

- Define a blank as a well-formed formula that is true by definition.
- Define the positive and negative areas for each Boolean operator.
- Show that each of Gentzen's rules is a derived rule of inference in terms of Peirce's rules.

Then any proof by Gentzen's rules is a proof by Peirce's rules.

Theoretical Issues

Peirce's rules have some remarkable properties:

- **Simplicity: Each rule inserts or erases a graph or subgraph.**
- **Symmetry: Each rule has an exact inverse.**
- **Depth independence: Rules depend on the positive or negative areas, not on the depth of nesting.**

They allow short proofs of remarkable theorems:

- **Reversibility Theorem. Any proof from *p* to *q* can be converted to a proof of *~p* from *~q* by negating each step and reversing the order.**
- **Cut-and-Paste Theorem. Any proof on a blank sheet may be "cut out" and "pasted in" any unshaded area nested arbitrarily deep.**
- **Resolution and natural deduction: Any proof by resolution may be converted to a proof by Peirce's version of natural deduction by negating each step and reversing the order.**

For proofs of these theorems and further discussion of the issues, see Section 6 of http://www.jfsowa.com/pubs/egtut.pdf

A Problem in Automated Reasoning

Larry Wos (1988), a pioneer in automated reasoning methods, stated 33 unsolved problems. His problem 24:

> ***Is there a mapping between clause representation and natural-deduction representation (and corresponding inference rules and strategies) that causes reasoning programs based respectively on the two approaches or paradigms to attack a given assignment in an essentially identical fashion?***

The answer in terms of Peirce's rules is yes:

- **The inference rules for Gentzen's clause form and natural deduction are derived rules of inference in terms of the EG rules.**
- **Any proof in clause form (by resolution) may be converted, step by step, to a proof by EG rules.**
- **Any such proof may be converted to a proof by Peirce's version of natural deduction by negating each step and reversing the order.**
- **Convert that proof by Peirce's rules to a proof by Gentzen's rules.**

Alpha, Beta, and Gamma Graphs

Peirce classified EGs in three categories:

- **Alpha graphs use only conjunction and negation to represent propositional logic.**
- **Beta graphs add the existential quantifier to represent full FOL.**
- **Gamma graphs extend EGs with metalanguage, modal logic, and higher-order logic.**

The semantics of CLIP and any EG translated to or from CLIP is defined by ISO standard 24707 for Common Logic (CL).

- **For Alpha and Beta graphs, CL model theory is consistent with Peirce's model theory, which he called *endoporeutic*.**
- **CL semantics also supports quantification over relations in a way that is compatible with Peirce's version.**
- **But extensions to CLIP are needed for other Gamma features.**

For details, see Section 5 of http://www.jfsowa.com/pubs/eg2cg.pdf

Psychology

Endorsement by the psychologist Philip Johnson-Laird (2002):

> **"Peirce's existential graphs... establish the feasibility of a diagrammatic system of reasoning equivalent to the first-order predicate calculus."**
>
> **"They anticipate the theory of mental models in many respects, including their iconic and symbolic components, their eschewal of variables, and their fundamental operations of insertion and deletion."**
>
> **"Much is known about the psychology of reasoning... But we still lack a comprehensive account of how individuals represent multiply-quantified assertions, and so the graphs may provide a guide to the future development of psychological theory."**

Johnson-Laird published many papers about mental models.

His comments on that topic are significant, especially in combination with the other properties of the graphs.

Reasoning with Mental Models

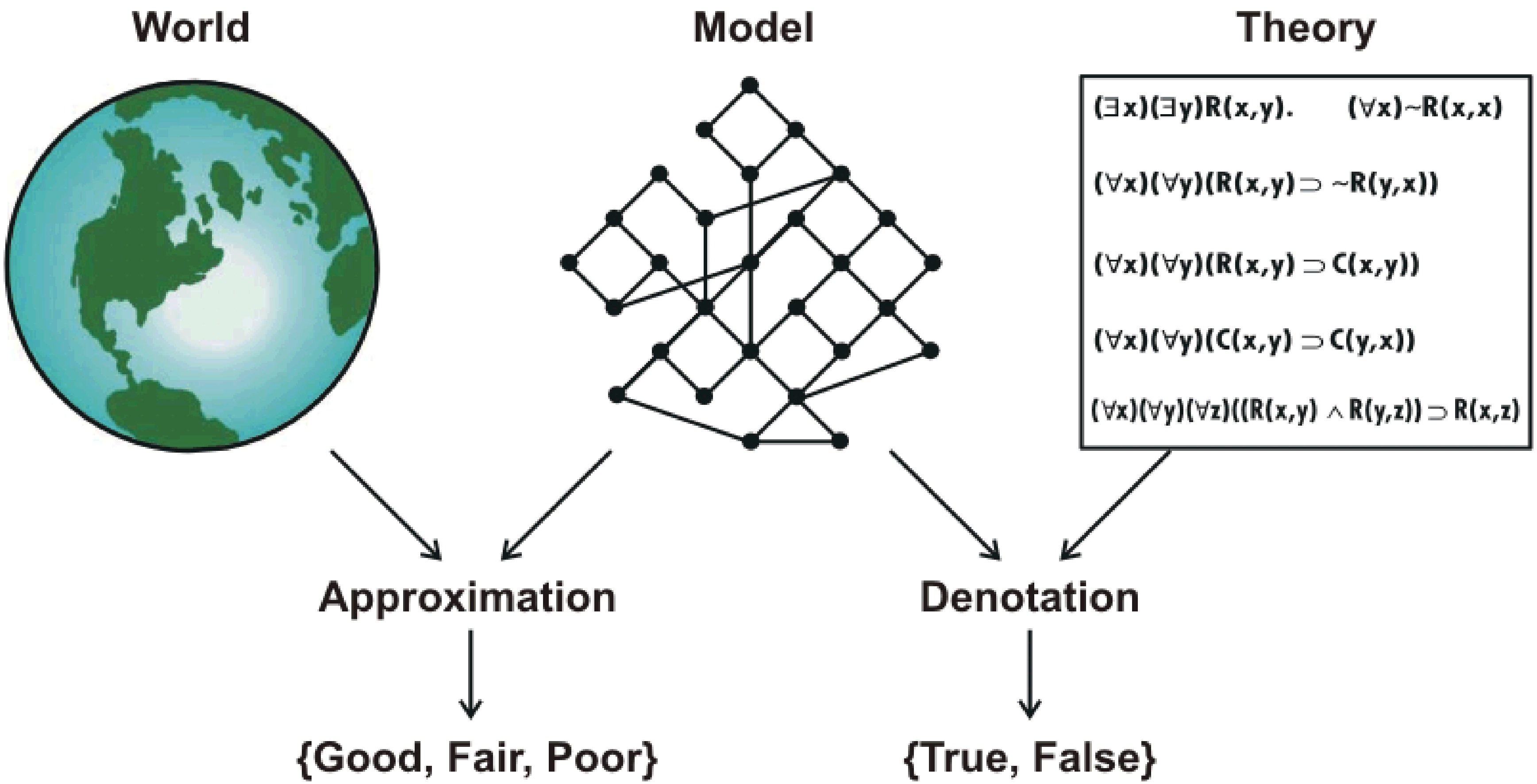
From Damasio and other neuroscientists:

- Mental models are patterns in the sensory projection areas that resemble patterns generated during perception.
- But the stimuli that generate mental models come from the frontal lobes, not from sensory input.
- The content of the mental models is generated by assembling fragments of earlier perceptions in novel combinations.

From suggestions by Johnson-Laird:

- The nodes of an existential graph could represent images or fragments of images from long-term memory.
- The connecting lines of an EG would show how those fragments are assembled to form a mental model.
- The logical features of EGs could be used to represent rules and constraints for reasoning about those models.

Models of Worlds, Real or Possible

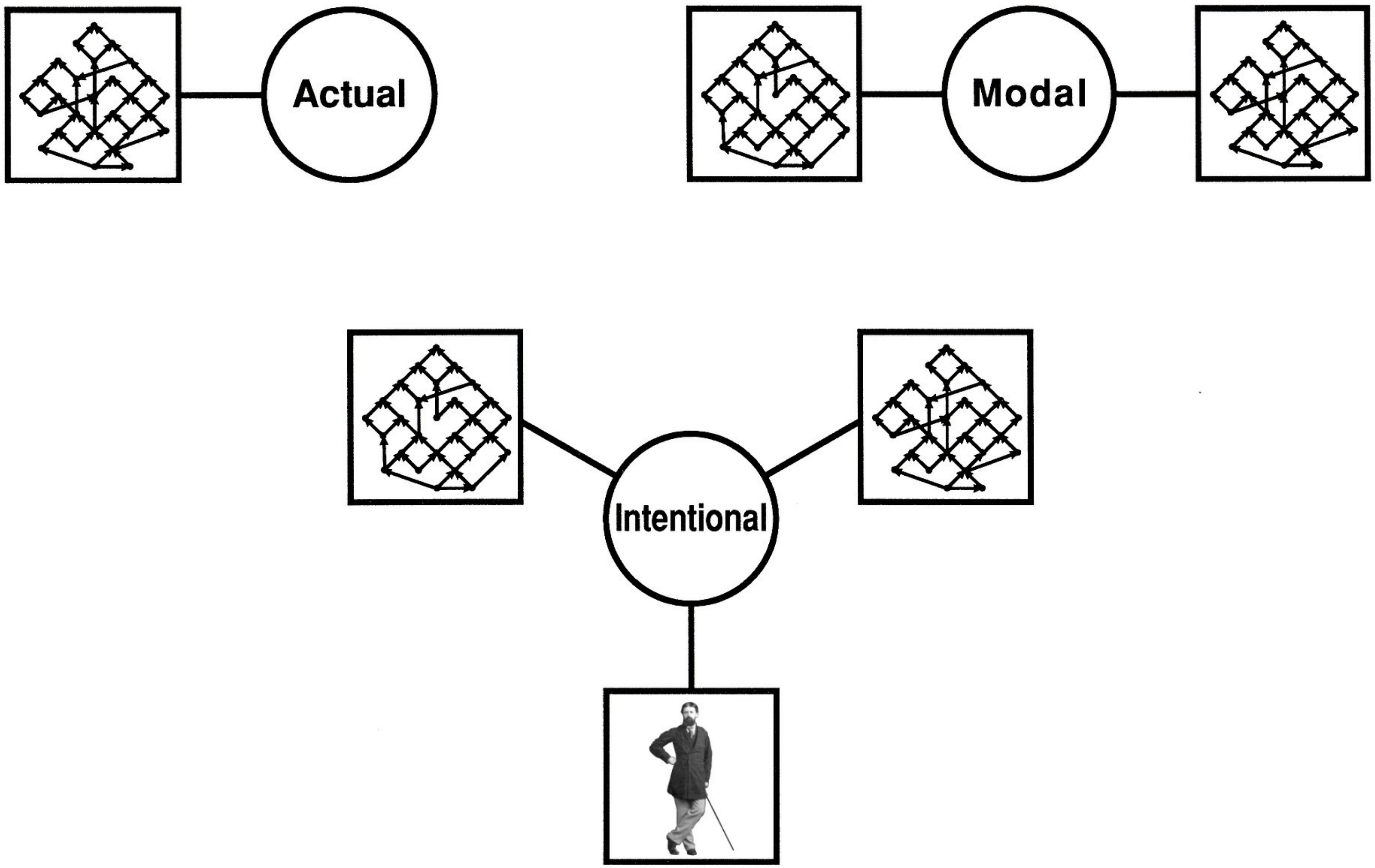


A Tarski-style model evaluates axioms of a theory in terms of a world, which may be described by a set, a network, or a database of facts.

For modal logic, the model may consist of a family of possible worlds.

In computer applications, possible worlds are represented by sets of propositions that are true (facts) or necessarily true (laws).

Actual, Modal, and Intentional Contexts



Three kinds of contexts, according to the source of knowledge:

- **Actual: Something factual about the world.**
- **Modal: Something possible, as determined by some hypothesis.**
- **Intentional: Something an agent believes, desires, or intends.**

Nested Situations

The three situations may be described as actual, modal, or intentional.

1. Actual: *Pierre is thinking of Marie, who is thinking of him.*

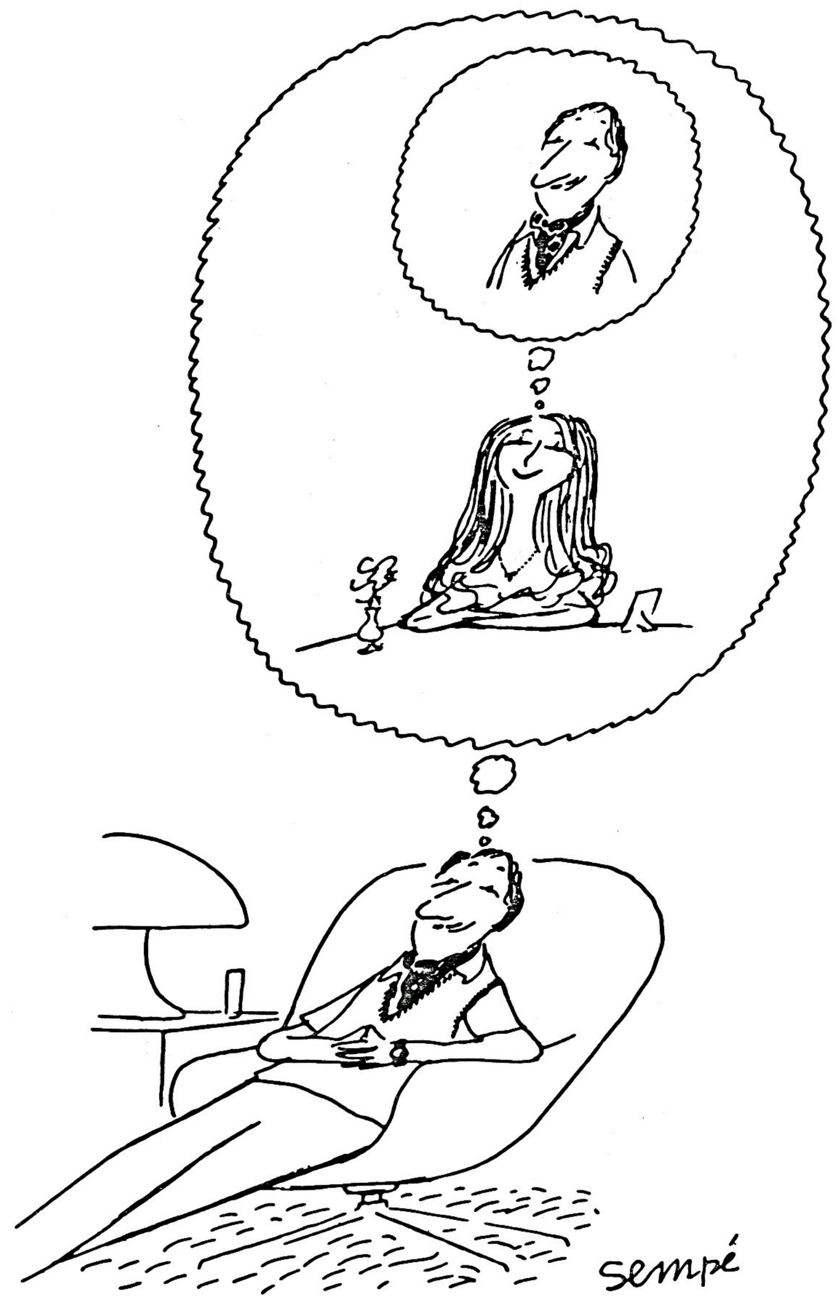
2. Modal: *Pierre is thinking of Marie, who may be thinking of him.*

3. Intentional: *Pierre hopes that Marie is thinking of him.*

In #1, both clauses are true, but Pierre may not know what Marie thinks.

In #2, the first clause is true, but the second may be true or false.

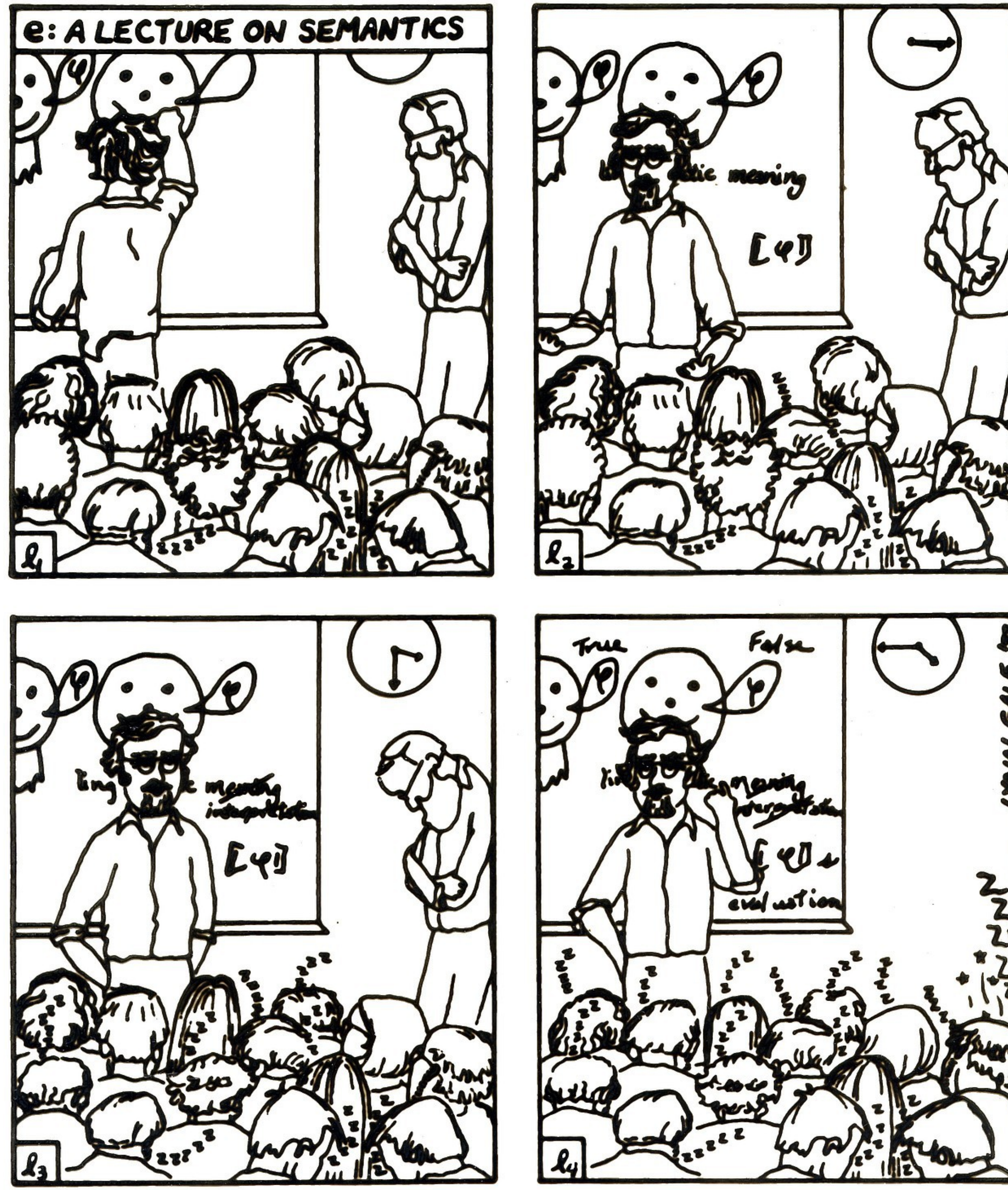
In #3, Pierre assumes or wishes that his thought is true, but it may be false.



In the situation *e*, John Perry is lecturing while Jon Barwise is standing on the right.

A language expression φ is a relation between a *discourse situation d*, a *speaker connection function c*, and a *described situation e*:
$d, c \,\|\varphi\|\, e$.

If φ is the expression *"the number of sleeping students"*, its value is 3 at 3:01 pm, 5 at 3:15, 9 at 3:30, and 19 at 3:45.



Mental Maps, Images, and Models

Observation by the neuroscientist Antonio Damasio (2010):

> ***"The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them."***

The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires.

Words and phrases of language can be generated from them.

They provide a "model theoretic" semantics for language that uses perception and action for testing models against reality.

Like Tarski's models, they define the criteria for truth, but they are flexible, dynamic, and situated in the daily drama of life.

Related Readings

Sowa, John F. (2011) Peirce's tutorial on existential graphs, http://www.jfsowa.com/pubs/egtut.pdf

Sowa, John F. (2013) From existential graphs to conceptual graphs, http://www.jfsowa.com/pubs/eg2cg.pdf

Sowa, John F. (2015) Slides for a tutorial on natural logic, http://www.jfsowa.com/talks/natlog.pdf

Johnson-Laird, Philip N. (2002) Peirce, logic diagrams, and the elementary operations of reasoning, *Thinking and Reasoning* 8:2, 69-95. http://mentalmodels.princeton.edu/papers/2002peirce.pdf

Pietarinen, Ahti-Veikko (2009) Peirce's development of qantification theory, http://www.helsinki.fi/peirce/PEA/Pietarinen%20%2d%20Peirce%27s%20Development.pdf

Pietarinen, Ahti-Veikko (2003) Peirce's magic lantern of logic: Moving pictures of thought, http://www.helsinki.fi/science/commens/papers/magiclantern.pdf

Pietarinen, Ahti-Veikko (2011) Moving pictures of thought II, *Semiotica* 186:1-4, 315–331, http://www.helsinki.fi/~pietarin/publications/Semiotica-Diagrams-Pietarinen.pdf

Sowa, John F. (2010) Role of logic and ontology in language and reasoning, http://www.jfsowa.com/pubs/rolelog.pdf

Sowa, John F. (2006) Peirce's contributions to the 21st Century, http://www.jfsowa.com/pubs/csp21st.pdf

ISO/IEC standard 24707 for Common Logic, http://standards.iso.org/ittf/PubliclyAvailableStandards/c039175_ISO_IEC_24707_2007(E).zip

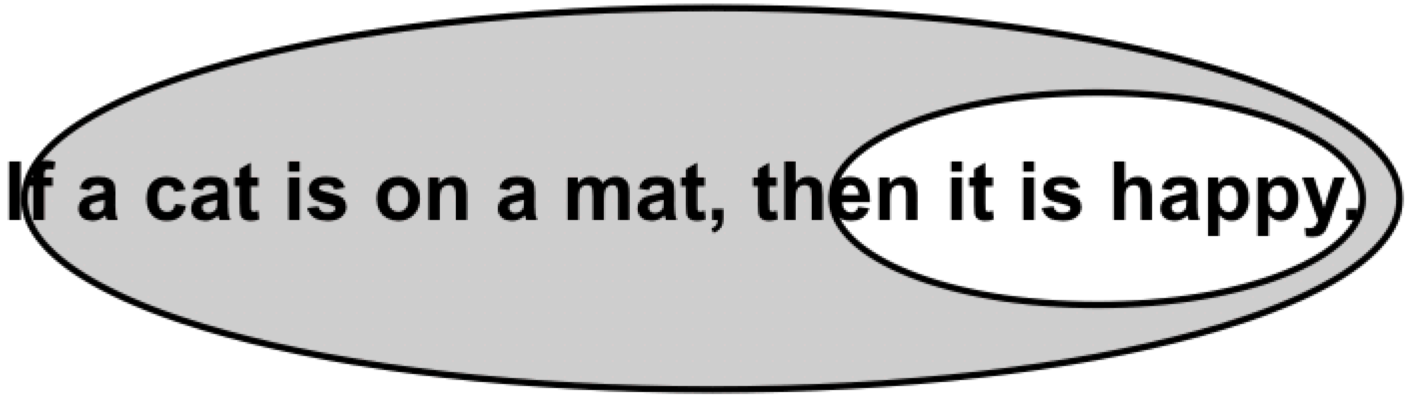
For other references, see the general bibliography, http://www.jfsowa.com/bib.htm

Peirce and Kamp

Peirce and Kamp independently developed isomorphic logics, while they were working on different, but related problems:

- **Peirce considered logic to be an integral part of semiotics, and he extended logic to include all methods of reasoning with signs.**
- **Peirce was also employed as an associate editor of the *Century Dictionary*, for which he wrote or edited over 16,000 definitions.**
- **Kamp developed a version of logic that had a simple mapping to and from natural languages.**
- **Peirce and Kamp had common interests, which led to some convergence in their choice of representations.**
- **Kamp and his colleagues showed how the DRS (or EG) notations could facilitate research in linguistics, especially anaphora.**
- **Peirce discovered very general rules of inference that could be applied to a wide range of notations, including EG and DRS.**
- **Their results are complementary.**

Applying Peirce's Rules to English



Start with any English sentence that can be mapped to or from a DRS:

- **Draw ovals around negative areas.**
- **Draw the ovals through words like *not, if, then, every, either, or*.**
- **Shade negative areas, and leave positive areas unshaded.**

A generalization of Peirce's first pair of rules:

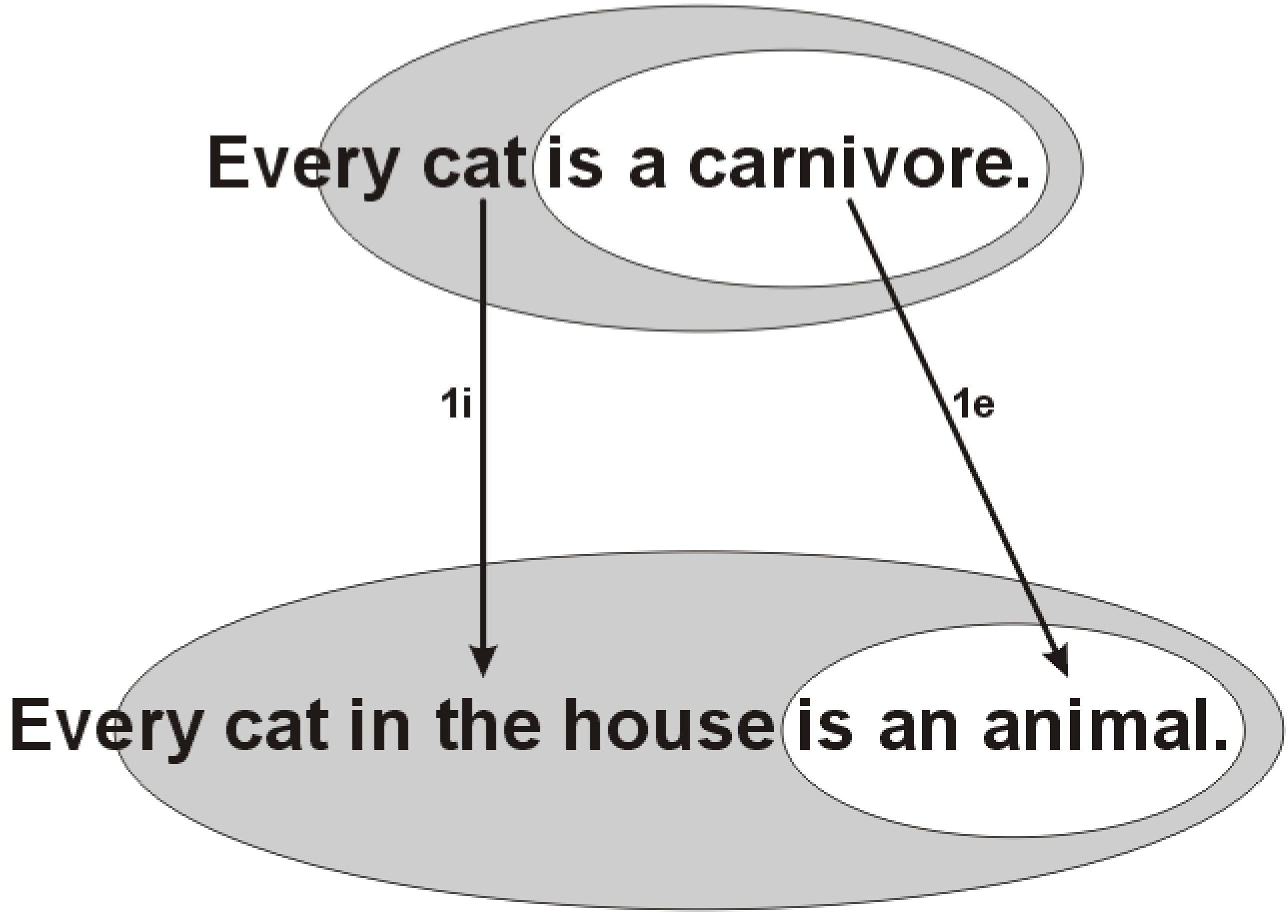
- **Insert: In a negative context (shaded), any propositional expression may be replaced by a more specialized expression.**
- **Erase: In a positive context (unshaded), any propositional expression may be replaced by a more general expression.**

An Inference in English

Use shading to mark the positive and negative parts of each sentence.

Rule 1i specializes 'cat' to 'cat in the house'.

Rule 1e generalizes 'carnivore' to 'animal'.



This method of reasoning is sound for sentences that can be mapped to a formal logic. It can also be used on propositional parts of sentences that contain some nonlogical features.

Icons and Diagrams

Three kinds of signs: Icons, indexes, and symbols.

- An icon has a structural resemblance to its referent.
- An index points to its referent by some kind of connection.
- A symbol indicates its referent by some habit or convention.

Algebraic notations combine symbols and indexes with linear icons for the operators and transformation rules.

Diagrams combine 2-D icons with symbols and indexes:

- A road map is an icon of a road system with symbolic labels.
- A topographic map is a labeled icon of some land surface.
- A map of roads plus topography can combine both.

Peirce’s existential graphs (EGs) are logic diagrams.

- Icons represent existence, identity, conjunction, negation, and scope.
- But generalized EGs can also use icons to represent relations.
- Perception can make implicit information explicit.

Implicit Information in an Icon

To answer a question by looking at the map:

- **Which country is closer to Africa: Canda or the USA?**
- **Observe the two congruent lines, which are determined by 4 data points.**
- **The possible observations grow as D^n — where D is the number of data points in the icon, and n is the number used in the observation.**

The information implicit in an icon can be far greater than the information explicitly encoded in symbols and indexes.

Euclid’s Drawings in an EG Proof

Euclid stated his proofs in procedural form:

- **His proofs mix imperative statements about what to draw with declarative statements about the result.**

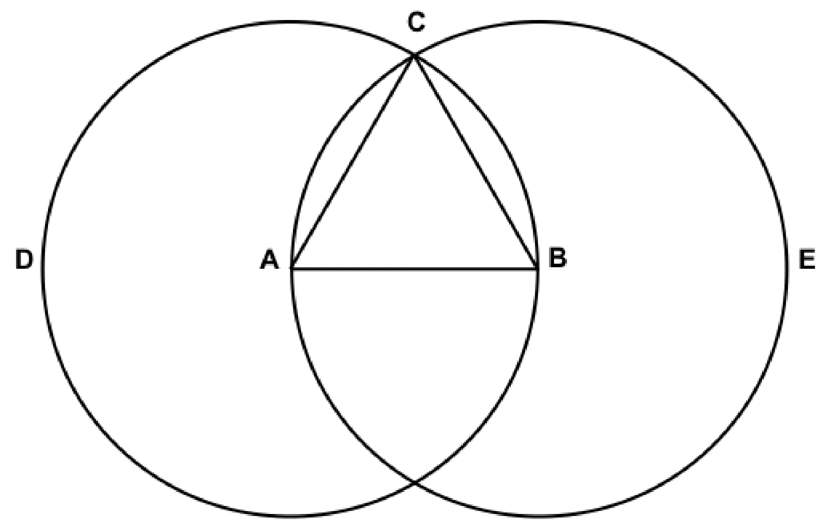
To simplify the logic, use only declarative sentences:

- **Euclid’s Proposition 1, as translated by Thomas Heath:**
  On a given finite straight line, to draw an equilateral triangle.
- **Restate that proposition as a conditional:**
  If there is a finite straight line AB, then there is an equlilateral triangle with AB as one of its sides.
- **Treat every imperative sentence beginning with *let* as a directive for the next step in carrying out the proof.**

When a drawing appears in an EG, represent lines of identity in a color that is not used in the diagrams.

Euclid’s naming conventions may be used to represent lines of identity in EGs.

Euclid’s Proposition 1



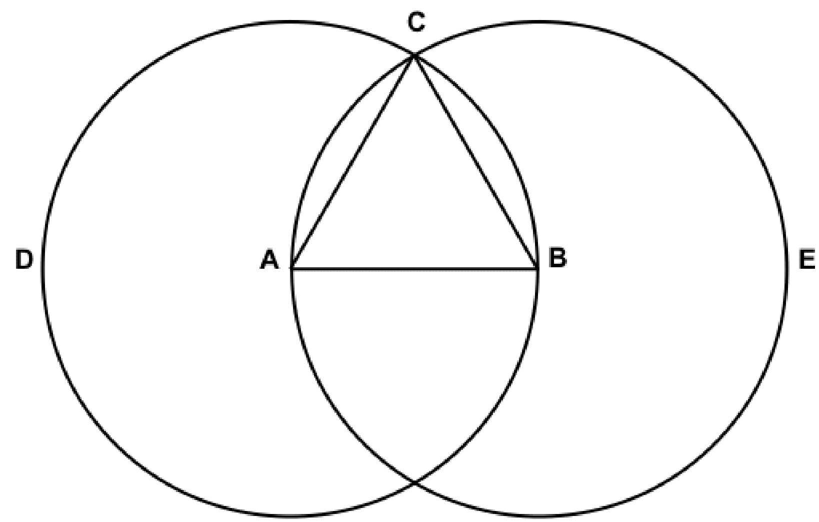
Euclid’s statement, as translated by Thomas Heath:

- **On a given finite straight line, to draw an equilateral triangle.**

The creative insight is to draw two circles:

- **The circle with center at A has radii AB and AC.**
- **The circle with center at B has radii BA and BC.**
- **Since all radii of a circle have the same length, the three lines AB, AC, and BC form an equilateral triangle.**

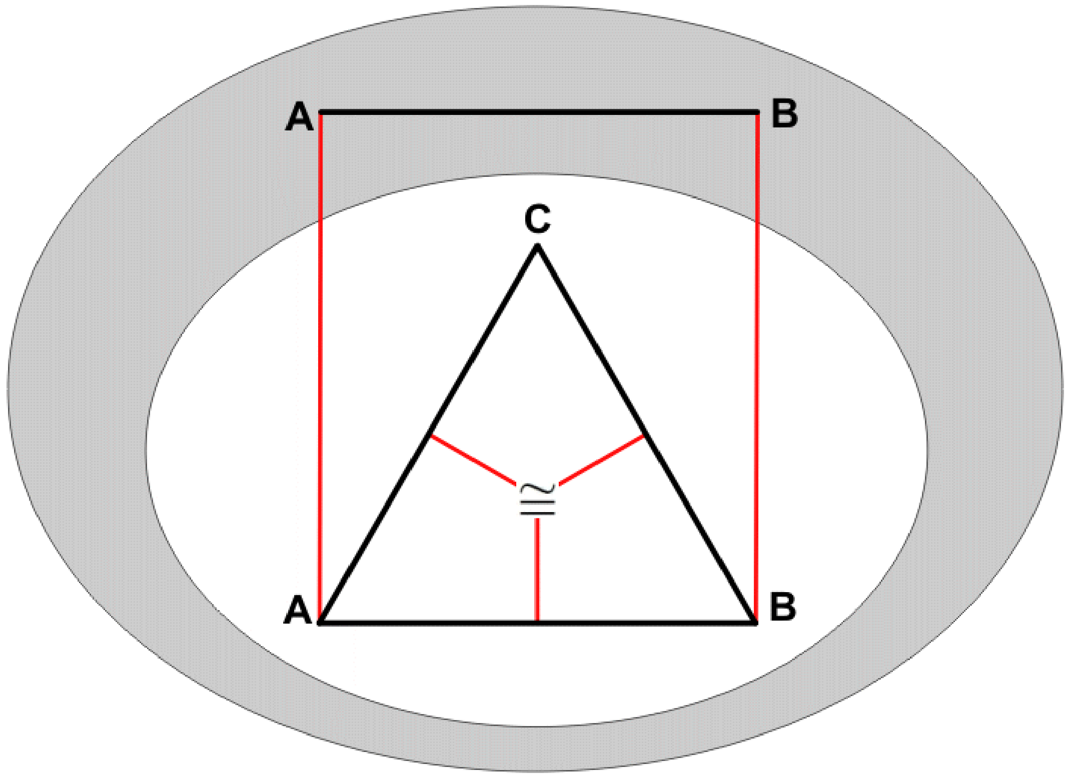
Linear Notations Obscure the Insight



On a given finite straight line, to draw an equilateral triangle.

Let AB be the given finite straight line. Thus it is required to construct an equilateral triangle on the straight line AB. With center A and distance AB, let the circle BCD be described [Postulate 3]. Again with center B and distance BA, let the circle ACE be described [Post. 3]. And from the point C, in which the circles cut one another, to the points A, B let the straight lines CA, CB be joined [Post. 1]. Now, since the point A is the center of the circle CDB, AC is equal to AB [Definition 15]. Again, since the point B is the center of the circle CAE, BC is equal to BA [Def. 15]. But CA was also proved equal to AB. Therefore, each of the straight lines CA, CB is equal to AB. And things which are equal to the same thing are also equal to one another [Common Notion 1]. Therefore, CA is also equal to CB. Therefore, the three straight lines CA, AB, BC are equal to one another. Therefore, the triangle ABC is equilateral, and it has been constructed on the given finite straight line. QED

Proposition 1 as a Generalized EG



English: ***If there is a finite straight line AB, then there is an equlilateral triangle with AB as one of its sides.***

The symbol '≅' names a relation that states that the three sides of the triangle are congruent.

The lines of identity are in red. The lines from A to A and B to B are redundant and may be omitted.

Observation and Imagination

Peirce's rules may be applied to icons in generalized EGs:

- **An icon in GEG asserts a pattern of relations among its parts.**
- **A line of identity (or a sign that serves as an index) may be used to link some part of an icon to an EG, GEG, or another icon.**
- **The GEG rules of inference may insert or erase icons or parts of icons according to the same rules for EGs or parts of EGs.**

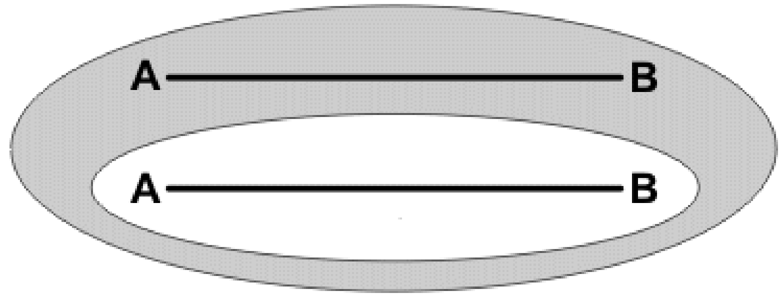
Two new rules of inference for relating icons to GEGs:

- **Observation: Any GEG that is implied by an icon may be inserted in the same area as that icon.**
- **Imagination: Any icon that is implied by a GEG may be inserted in the same area as that GEG.**

The rules of inference for GEGs are sound. They preserve truth because no rule adds information that is not implicit in the GEG.

But the rules may make implicit information explicit.

Proving Proposition 1



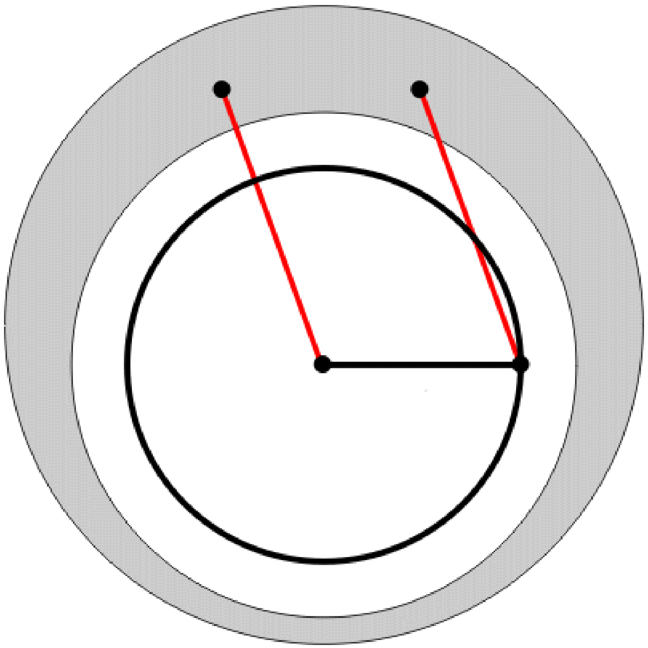
The first steps of the proof in EG form:

- Begin with the empty graph — a blank sheet of paper.
- By rule 3i, insert a double negation around the blank.
- By rule 1i, insert the original hypothesis into the shaded area.
- By rule 2i, iterate (copy) the hypothesis into the unshaded area.
- Conclusion: If there is a line AB, then there is a line AB.

Continuation of the proof:

- By repeated use of rule 2i, copy axioms and definitions into the unshaded area and use other rules to apply them in that area.
- Finally, the condition of Proposition 1 remains in the shaded area and the complete diagram of Proposition 1 appears in the unshaded area.

Euclid's Postulate 3



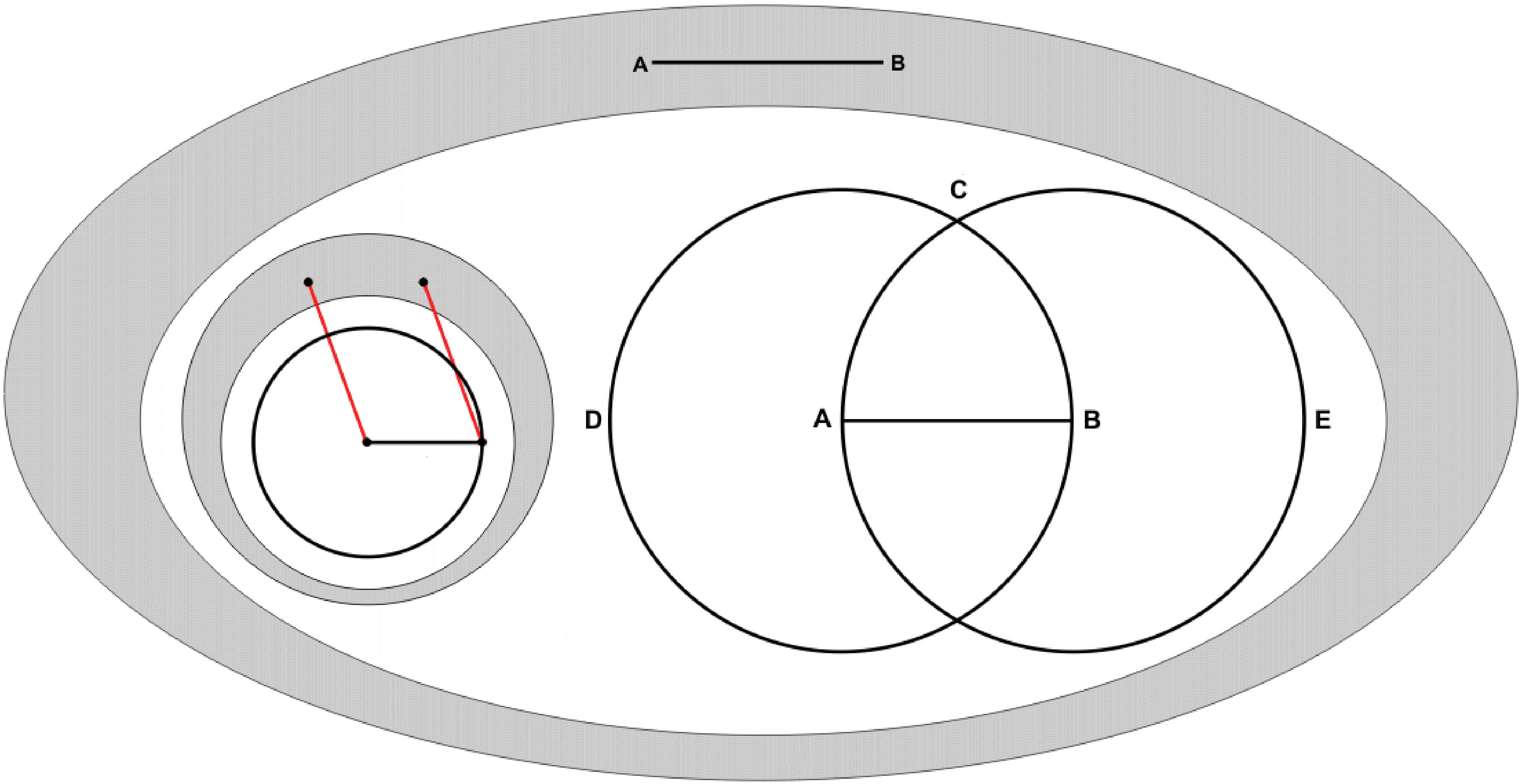
Euclid: ***To describe a circle with any center and distance.***

English for the EG: ***If there are two points, then there exists a circle with one point at its center and a radius from the center to the other point.***

Next steps in proving Proposition 1:

- **By rule 2i, insert Postulate 3 into the unshaded area of the EG in slide 26.**
- **Then use modus ponens, a derived rule of inference from Peirce's rules.**

Continuation

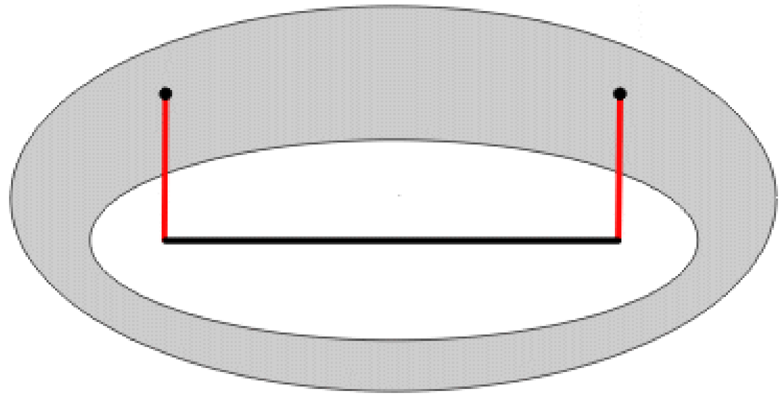


By modus ponens, insert a circle with center A and radius AB.

By modus ponens, insert a circle with center B and radius BA.

Then by rule 1e, erase Postulate 3 from the unshaded area.

Euclid's Postulate 1



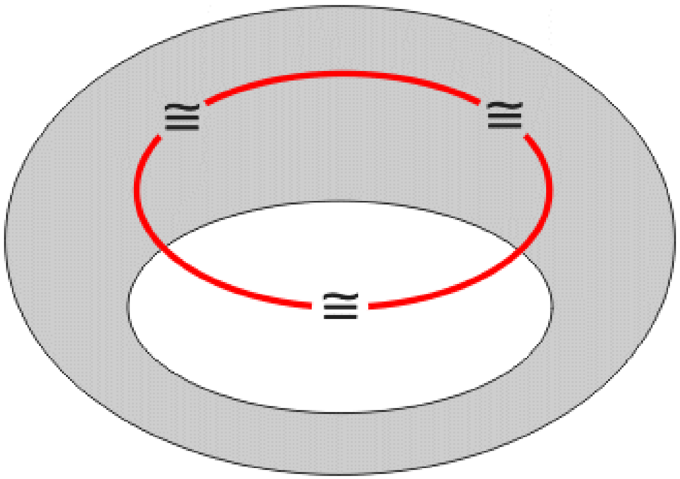
Euclid: ***To draw a straight line from any point to any point.***

English for the EG: ***If there are two points, then there exists a straight line from one to the other.***

Continuation:

- **By rule 2i, iterate this EG into the unshaded area of the EG in slide 38.**
- **By modus ponens, use it to insert a line from C to A.**
- **By modus ponens, insert another line from C to B.**
- **By rule 1e, erase Postulate 1 in the unshaded area.**
- **By the definition of triangle, lines AB, CA, and CB form a triangle ABC.**

Euclid's Common Notion 1



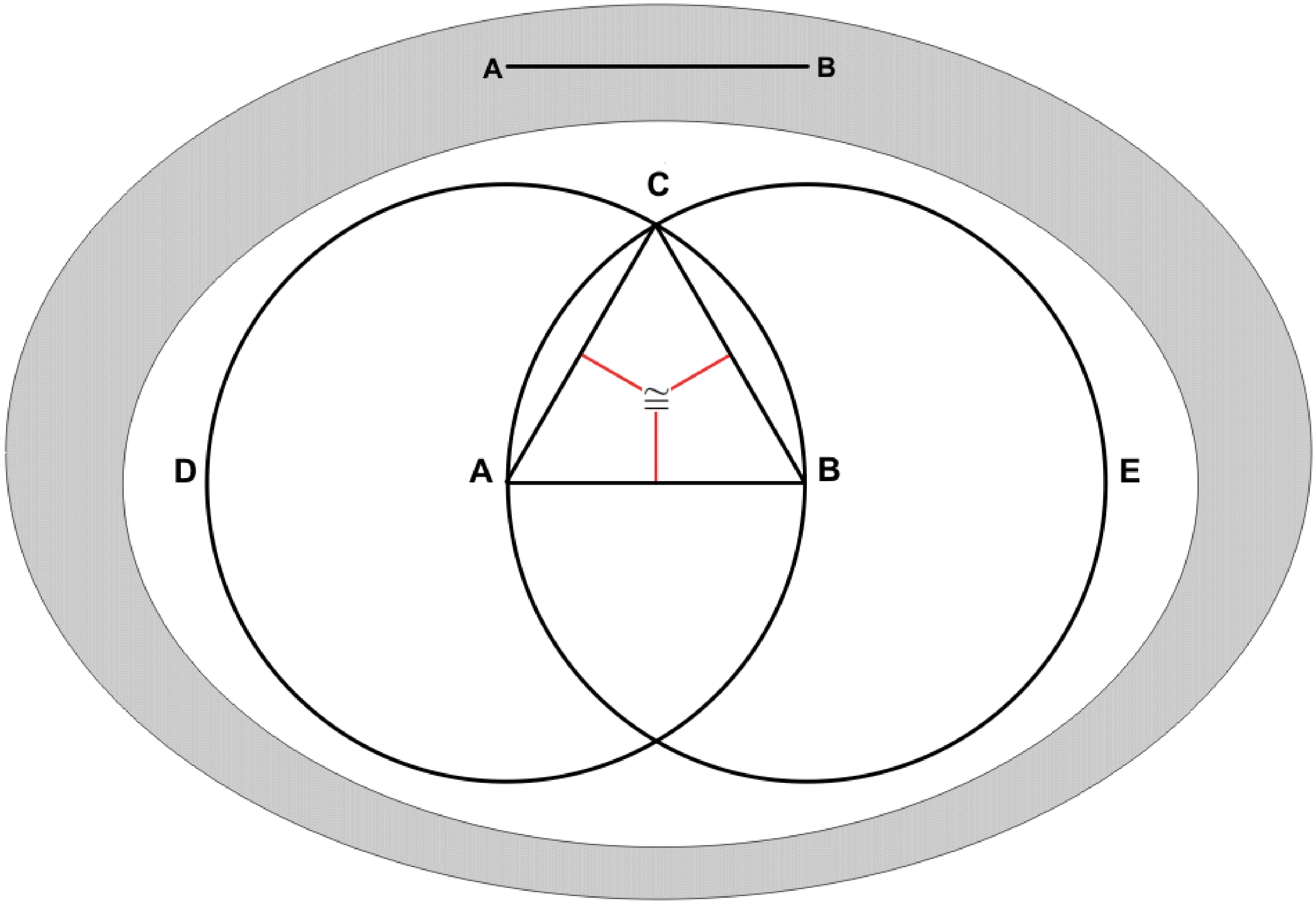
Euclid: ***Things which are equal to the same thing are also equal to one another.***

English for the EG: ***If two things are congruent to a third, then they are congruent to each other.***

Concluding steps:

- **By rule 2i, iterate this EG into the unshaded area of the EG.**
- **By definition 15, the radii AC and AB of the circle CDB are congruent.**
- **By definition 15, the radii BC and BA of the circle CAE are congruent.**
- **By common notion 1, the lines AC, AB, and BC are congruent.**
- **By rule 1e, erase Common Notion 1 from the unshaded area.**

Quod Erat Demonstrandum



This EG is the result of carrying out every step in Euclid's proof.

By rule 1e, erase the circles to derive the EG for Proposition 1.

The Role of Icons in Logic

As these examples show, icons can make logic more readable.

But icons can play a more fundamental role:

- **The visual images used for induction and abduction can also be integral components of the notations used in deduction.**
- **The implicit information in icons is potentially much larger than the explicit information in the symbols of a linear notation.**
- **That information can support heuristics for selecting appropriate axioms or background knowledge.**
- **The open-ended amount of information can make a logic with icons more expressive than a logic without icons.**

Icons represent relations just as well or better than symbols:

- **Icons show sensory patterns more precisely and in more detail.**
- **The rule of observation maps iconic patterns to symbolic relations.**
- **The rule of imagination may map the symbolic relations to an icon that is identical to the original, but more likely to less detailed icons.**
- **A combination of icons and symbols can represent a broader range of human knowledge than either kind by itself.**

Theoretical Issues

Peirce's rules have some remarkable properties:

- **Simplicity: Each rule inserts or erases a graph or subgraph.**
- **Symmetry: Each rule has an exact inverse.**
- **Depth independence: Rules depend on the positive or negative areas, not on the depth of nesting.**

They allow short proofs of remarkable theorems:

- **Reversibility Theorem. Any proof from *p* to *q* can be converted to a proof of ~*p* from ~*q* by negating each step and reversing the order.**
- **Cut-and-Paste Theorem. If *q* can be proved from *p* on a blank sheet, then in any positive area where *p* occurs, *q* may be substituted for *p*.**
- **Resolution and natural deduction: Any proof by resolution can be converted to a proof by Peirce's version of natural deduction by negating each step and reversing the order.**

For proofs of these theorems and further discussion of the issues, see Section 6 of http://www.jfsowa.com/pubs/egtut.pdf

Natural Deduction

Gerhard Gentzen developed a proof procedure that he called *natural deduction*.

But Peirce's method is a version of natural deduction that is simpler and more general than Gentzen's:

Peirce's Method	Gentzen's Method
6 rules	16 rules
3 symmetric pairs	Many irregularities
Simple operations	Requires provability
Straight-line proofs	Complex bookkeeping
Date: 1897-1909	Date: 1935

For details, see pp. 24 to 26 of http://www.jfsowa.com/pubs/egtut.pdf

Gentzen’s Natural Deduction

	Introduction Rules	Elimination Rules
$\land$	$\dfrac{A,\ B}{A \land B}$	$\dfrac{A \land B}{A}$ $\quad$ $\dfrac{A \land B}{B}$
$\lor$	$\dfrac{A}{A \lor B}$ $\quad$ $\dfrac{B}{A \lor B}$	$\dfrac{A \lor B,\ \ A \vdash C,\ \ B \vdash C}{C}$
$\supset$	$\dfrac{A \vdash B}{A \supset B}$	$\dfrac{A,\ \ A \supset B}{B}$
$\sim$	$\dfrac{A \vdash \bot}{\sim A}$ $\quad$ $\dfrac{\bot}{A}$	$\dfrac{A,\ \ \sim A}{\bot}$ $\quad$ $\dfrac{\sim\sim A}{A}$
$\forall$	$\dfrac{A(a)}{(\forall x)A(x)}$	$\dfrac{(\forall x)A(x)}{A(t)}$
$\exists$	$\dfrac{A(t)}{(\exists x)A(x)}$	$\dfrac{(\exists x)A(x),\ \ A(a) \vdash B}{B}$

Like Peirce, Gentzen assumed only one axiom: a blank sheet of paper.

But Gentzen had more operators and more complex, nonsymmetric pairs of rules for inserting or erasing operators.

Role of the Empty Sheet

Both Peirce and Gentzen start a proof from an empty sheet.

In Gentzen's syntax, a blank sheet is not a well-formed formula.

- **Therefore, no rule of inference can be applied to a blank.**
- **A method of making and discharging an assumption is the only way to begin a proof.**

But in EG syntax, an empty graph is a well-formed formula.

- **Therefore, a double negation may be drawn around a blank.**
- **Then any assumption may be inserted in the negative area.**

Applying Peirce's rules to predicate calculus:

- **Define a blank as a well-formed formula that is true by definition.**
- **Define the positive and negative areas for each Boolean operator.**
- **Show that each of Gentzen's rules is a derived rule of inference in terms of Peirce's rules.**

Then any proof by Gentzen's rules is a proof by Peirce's rules.

A Problem in Automated Reasoning

Larry Wos (1988), a pioneer in automated reasoning methods, stated 33 unsolved problems. His problem 24:

> ***Is there a mapping between clause representation and natural-deduction representation (and corresponding inference rules and strategies) that causes reasoning programs based respectively on the two approaches or paradigms to attack a given assignment in an essentially identical fashion?***

The answer in terms of Peirce's rules is yes:

- **The inference rules for Gentzen's clause form and natural deduction are derived rules of inference in terms of the EG rules.**
- **Any proof in clause form (by resolution) can be converted, step by step, to a proof by EG rules.**
- **Any such proof can be converted to a proof by Peirce's version of natural deduction by negating each step and reversing the order.**
- **Convert the proof by Peirce's rules to a proof by Gentzen's rules.**

Psychological Issues

Endorsement by the psychologist Philip Johnson-Laird (2002):

> ***Peirce's existential graphs... anticipate the theory of mental models in many respects, including their iconic and symbolic components, their eschewal of variables, and their fundamental operations of insertion and deletion.***
>
> ***Much is known about the psychology of reasoning... But we still lack a comprehensive account of how individuals represent multiply-quantified assertions, and so the graphs may provide a guide to the future development of psychological theory. ****

Johnson-Laird published a book on mental models.

His comments on that topic are significant, and the option of using icons in generalized EGs strenghthens the claim.

*** See http://mentalmodels.princeton.edu/papers/2002peirce.pdf**

Mental Maps, Images, and Models

The neuroscientist Antonio Damasio (2010):

> ***The distinctive feature of brains such as the one we own is their uncanny ability to create maps... But when brains make maps, they are also creating images, the main currency of our minds. Ultimately consciousness allows us to experience maps as images, to manipulate those images, and to apply reasoning to them.***

The maps and images form mental models of the real world or of the imaginary worlds in our hopes, fears, plans, and desires.

Words and phrases of language can be generated from them.

They provide a “model theoretic” semantics for language that uses perception and action for testing models against reality.

With Generalized EGs, the model theory can be based on direct mappings between icons in the logic and icons in the model.

Reasoning with Mental Models

From Damasio and other neuroscientists:

- Mental models are patterns in the sensory projection areas that resemble patterns generated during perception.
- But the stimuli that generate mental models come from the frontal lobes, not from sensory input.
- The content of the mental models is generated by assembling fragments of earlier perceptions in novel combinations.

From suggestions by Johnson-Laird:

- The nodes of an existential graph could represent images or fragments of images from long-term memory.
- The connecting lines of an EG would show how those fragments are assembled to form a mental model.
- The logical features of EGs could be used to represent rules and constraints for reasoning about those models.

Computability

Can images enable proofs with Generalized EGs to go beyond what can be computed with a Turing machine?

With the kinds of diagrams Euclid used, no:

- **Every diagram has a finite number of figures, each determined by a finite number of points in a finite number of possible patterns.**
- **Euclid's proofs can be mapped to EGs and then to CLIP.**
- **EG proofs may be more readable, but no more powerful.**

But with continuous images, maybe:

- **Continuous images may have an uncountable infinity of points, shapes, relationships, and possible transformations.**
- **They would enable an open-ended variety of views or perspectives at different distances, angles, and magnifications.**
- **Repeated observations could extract an unlimited amount of information from them.**

Turing Oracle Machines

How could Generalized EGs go beyond a Turing machine?

Turing a-machines can represent any algorithm:

- **They can perform any computation by any digital computer.**
- **But they do not allow any external input during a computation.**

But Turing (1939) also discussed oracles for o-machines:

- **An o-machine is an a-machine that can interrogate an oracle.**
- **Emil Post expanded Turing's one-page summary to show how an o-machine can extend the computational power of an a-machine.**
- **Robert Soare (2009) presented a detailed history and analysis of the Post-Turing hypothesis. ***
- **Soare claims that a digital computer connected to I/O devices can have the power of an o-machine.**
- **Generalized EGs with the option of extracting information from continuous images could also have the power of an o-machine.**

*** See Robert Soare, http://www.people.cs.uchicago.edu/~soare/History/turing.pdf**

Conclusion

Peirce called existential graphs "the logic of the future."

Computer graphics and virtual reality can implement them:

- **The icons in two-dimensional maps can be generalized to three dimensions and even 3+1 dimensions for motion and change.**
- **Conjunctions and lines of identity can be represented in any dimension.**
- **For negation, the ovals can be generalized to closed shapes in any number of dimensions. Rvachev functions can implement them.**
- **Viewers with VR goggles could wander through 4-dimensional EGs, watch the movies, and manipulate icons according to Peirce's rules.**

Peirce's claim is consistent with neuroscience:

- **As Damasio said, images are "the main currency of our minds."**
- **As Johnson-Laird observed, Peirce's rules of inference insert and erase graphs and subgraphs — operations that neural processes can perform.**
- **Generalized EGs can include arbitrary images in the graphs.**
- **When Peirce claimed that EGs represent "a moving picture of the action of the mind in thought," he may have imagined something similar.**

Related Readings

Sowa, John F. (2011) Peirce's tutorial on existential graphs, http://www.jfsowa.com/pubs/egtut.pdf

Sowa, John F. (2013) From existential graphs to conceptual graphs, http://www.jfsowa.com/pubs/eg2cg.pdf

Pólya, George (1954) *Mathematics and Plausible Reasoning*, Volume I: *Induction and Analogy in Mathematics*, Volume II: *Patterns of Plausible Inference*, Princeton: University Press.

Johnson-Laird, Philip N. (2002) Peirce, logic diagrams, and the elementary operations of reasoning, *Thinking and Reasoning* 8:2, 69-95. http://mentalmodels.princeton.edu/papers/2002peirce.pdf

Pietarinen, Ahti-Veikko (2003) Peirce's magic lantern of logic: Moving pictures of thought, http://www.helsinki.fi/science/commens/papers/magiclantern.pdf

Pietarinen, Ahti-Veikko (2011) Moving pictures of thought II, *Semiotica* 186:1-4, 315–331, http://www.helsinki.fi/~pietarin/publications/Semiotica-Diagrams-Pietarinen.pdf

Sowa, John F. (2010) Role of logic and ontology in language and reasoning, http://www.jfsowa.com/pubs/rolelog.pdf

Sowa, John F. (2006) Peirce's contributions to the 21st Century, http://www.jfsowa.com/pubs/csp21st.pdf

ISO/IEC standard 24707 for Common Logic, http://standards.iso.org/ittf/PubliclyAvailableStandards/c039175_ISO_IEC_24707_2007(E).zip

For other references, see the general bibliography, http://www.jfsowa.com/bib.htm