Capitolul 9. Funcţii de mai multe variabile.

1. **Să se determine şi să se reprezinte domeniile de definiţie ale următoarelor funcţii:**

1.1. $u = \sqrt{x} + y.$

1.2. $u = \sqrt{xy}.$

1.3. $u = \sqrt{4 - x^2 - y^2}.$

1.4. $u = \sqrt{x^2 + y^2 - 1}.$

1.5. $u = \sqrt{\dfrac{x^2}{9} + \dfrac{y^2}{4} - 1}.$

1.6. $u = \sqrt{(x^2 + y^2 - 4)(9 - x^2 - y^2)}.$

1.7. $u = \dfrac{1}{\sqrt{x^2 + y^2 - 16}}.$

1.8. $u = \dfrac{1}{\sqrt{9 - x^2 - y^2}}.$

1.9. $u = \sqrt{4 - x^2 - y^2} + \sqrt{x^2 + y^2 - 1}.$

1.10. $u = y\sqrt{1 - \cos x}.$

1.11. $u = \sqrt{\dfrac{x^2 + y^2 - x}{2x - x^2 - y^2}}.$

1.12. $u = \sqrt{\dfrac{x^2 + y^2 - y}{2y - x^2 - y^2}}.$

1.13. $u = \ln\left(1 - \dfrac{x^2}{9} - \dfrac{y^2}{16}\right).$

1.14. $u = \ln(x + y).$

1.15. $u = \sqrt{\ln(x^2 + y^2)}.$

1.16. $u = \lg\big(y^2 - 4x + 8\big).$

1.17. $u = \dfrac{\sqrt{4x - y^2}}{\ln(1 - x^2 - y^2)}.$

1.18. $u = \arcsin\dfrac{x}{y}.$

1.19. $u = \arccos\dfrac{y}{x + y}.$

1.20. $u = \arcsin\dfrac{x - 1}{y}.$

1.21. $u = \arcsin\dfrac{x}{y^2} + \arccos(1 - y).$

1.22. $u = \operatorname{ctg}[\pi(x + y)].$

1.23. $u = \sqrt{\sin[\pi(x^2 + y^2)]}.$

1.24. $u = \lg x - \ln\cos y.$

2. Să se studieze existenţa limitelor:

2.1. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2-y^2}{x^2+y^2}$.

2.2. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x-y}{x+y}$.

2.3. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2y^2}{x^2y^2+(y-x)^2}$.

2.4. $\lim\limits_{\substack{x\to 0\\ y\to 0}} y\sin\dfrac{1}{x}$.

2.5. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2+y^2}{|x|+|y|}$.

2.6. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{2xy}{x^2+y^2}$.

2.7. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x}{x+y}$.

2.8. $\lim\limits_{\substack{x\to 3\\ y\to 0}} \dfrac{\operatorname{tg} xy}{y}$.

3. Să se calculeze:

3.1. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{xy}{2-\sqrt{xy+4}}$.

3.2. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{\sqrt{xy+1}-1}{2xy}$.

3.3. $\lim\limits_{\substack{x\to 0\\ y\to 2}} \dfrac{\sin xy}{x}$.

3.4. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^4y^2+x^2y^4}{1-\cos\left(x^2+y^2\right)}$.

3.5. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \left(1+x^2+y^2\right)^{\frac{2}{x^2+y^2}}$.

3.6. $\lim\limits_{\substack{x\to \infty\\ y\to \infty}} \left(x^2+y^2\right)\sin\dfrac{1}{x^2+y^2}$.

3.7. $\lim\limits_{\substack{x\to \infty\\ y\to 0}} \left(1+\dfrac{1}{x}\right)^{\frac{x^2}{x+y}}$.

3.8. $\lim\limits_{\substack{x\to \infty\\ y\to \infty}} \dfrac{x^2+y^2}{e^{x+y}}$.

3.9. $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^3+y^3}{x^2+y^2}$.

3.10. $\lim\limits_{\substack{x\to \infty\\ y\to \infty}} \left(\dfrac{xy}{x^2+y^2}\right)^{y^2}$.

3.11. $\lim\limits_{\substack{x\to 1\\ y\to 0}} \dfrac{\ln^2(x+y)}{\sqrt{x^2+y^2-2x+1}}$.

3.12. $\lim\limits_{\substack{x\to \infty\\ y\to 3}} \left(1+\dfrac{1}{x}\right)^{\frac{2x^2}{x+y}}$.

4. Să se studieze continuitatea funcţiilor următoare în punctul $(0,0)$:

4.1. $f(x,y) = \begin{cases} \dfrac{xy}{(x^2+y^2)^2} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.2. $f(x,y) = \begin{cases} \dfrac{x-y}{(x+y)^3} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.3. $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.4. $f(x,y) = \begin{cases} xy^2 \cdot \dfrac{x^2-y^2}{x^2+y^2} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.5. $f(x,y) = \begin{cases} (x^2+y^2)\ln(x^2+y^2) & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.6. $f(x,y) = \begin{cases} \dfrac{2x^2y}{x^4+3y^2} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.7. $f(x,y) = \begin{cases} \dfrac{\sqrt{x^2+y^2}}{\sin xy} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

4.8. $f(x,y) = \begin{cases} \sin\dfrac{1}{x^2+y^2} & , \ x^2+y^2 \neq 0 \ , \\ 3 & , \ x=y=0 \ . \end{cases}$

4.9. $f(x,y) = \begin{cases} 3-x-y & , \ x^2+y^2 \neq 0 \ , \\ 5 & , \ x=y=0 \ . \end{cases}$

4.10. $f(x,y) = \begin{cases} \dfrac{x^3+y^3}{x^4+y^2} & , \ x^2+y^2 \neq 0 \ , \\ 0 & , \ x=y=0 \ . \end{cases}$

5. Să se calculeze derivatele parţiale de primul ordin ale următoarelor funcţii:

5.1. $f(x,y) = x^2 - 2xy + y^2 + 1.$

5.2. $f(x,y) = x^3 - 3x^2y + 2xy^2 + y^3.$

5.3. $f(x,y) = \dfrac{xy}{y-x}.$

5.4. $f(x,y) = \dfrac{x-y}{x+y}.$

5.5. $f(x,y) = \dfrac{x}{y}.$

5.6. $f(x,y) = \operatorname{arctg}\dfrac{x}{y}.$

5.7. $f(x,y) = \ln(x^2+y^2).$

5.8. $f(x,y) = x^2\cos y.$

5.9. $f(x,y) = e^{x^2y}.$

5.10. $f(x,y) = \ln\left(\sqrt{x} + \sqrt[3]{y}\right).$

5.11. $f(x,y) = x^y.$

5.12. $f(x,y) = xy + \dfrac{y}{x}.$

5.13. $f(x,y) = ye^{-xy}.$

5.14. $f(x,y) = \dfrac{x}{y} + \dfrac{y}{x}.$

5.15. $f(x,y) = \ln\dfrac{\sqrt{x^2+y^2}+x}{\sqrt{x^2+y^2}-x}.$

5.16. $f(x,y) = \ln\left(y + \sqrt{x^2+y^2}\right).$

5.17. $f(x,y) = \operatorname{arctg}\dfrac{x+y}{x-y}.$

5.18. $f(x,y) = \arcsin\dfrac{x+y}{xy}.$

5.19. $f(x,y) = \left(x^2+y^2\right)\operatorname{arctg}\dfrac{x}{y}.$

5.20. $f(x,y) = \arccos\dfrac{y}{\sqrt{x^2+y^2}}.$

5.21. $f(x,y) = \operatorname{arctg}\dfrac{x+y}{1-xy}.$

5.22. $f(x,y) = x^{y^2}.$

5.23. $f(x,y) = e^x\ln y + \sin y\ln x.$

5.24. $f(x,y) = \ln(x^2+y^2+3).$

5.25. $f(x,y,z) = (\cos x)^{yz}.$

5.26. $f(x,y,z) = xy + yz + xz.$

5.27. $f(x,y,z) = \sqrt{x^2+y^2+z^2}.$

5.28. $f(x,y,z) = y^{\frac{x}{z}}.$

5.29. $f(x,y,z) = \ln(1+x+y^2+z^3).$

5.30. $f(x,y,z) = \sin x\cos(yz).$

6. Să se calculeze derivatele parţiale de ordinul doi pentru următoarele funcţii:

6.1. $f(x,y) = x^3 + y^3 - 2x^2y + 3xy^2.$

6.2. $f(x,y) = xy + \dfrac{y}{x}.$

6.3. $f(x,y) = x^4 - x^3y + xy^2 - y^4.$

6.4. $f(x,y) = \dfrac{x}{\sin y^2}.$

6.5. $f(x,y) = y\cos(x-y).$

6.6. $f(x,y) = y^x.$

6.7. $f(x,y) = \operatorname{arctg}\dfrac{x+y}{1-xy}.$

6.8. $f(x,y) = \dfrac{x+y}{x-y}.$

6.9. $f(x,y) = \arccos(xy).$

6.10. $f(x,y) = \ln(e^x + e^y).$

6.11. $f(x,y) = \ln\left(x^2 + y^2\right).$

6.12. $f(x,y) = \sqrt[3]{x^2} + \sqrt[4]{y^3}.$

6.13. $f(x,y) = \operatorname{arctg}\dfrac{x+y}{y}.$

6.14. $f(x,y) = ye^x.$

6.15. $f(x,y) = e^y(\cos x + y\sin x).$

6.16. $f(x,y) = \dfrac{y^2}{1-2x}.$

6.17. $f(x,y) = \arcsin\dfrac{y}{\sqrt{x^2+y^2}}.$

6.18. $f(x,y) = e^{x^2y}.$

6.19. $f(x,y) = \operatorname{arcctg}\dfrac{y}{x}.$

6.20. $f(x,y) = \sqrt{x^2+y^2}.$

6.21. $f(x,y) = y\ln\dfrac{x}{y}.$

6.22. $f(x,y) = e^{x^2+y}.$

6.23. $f(x,y) = \left(x^2+y^2\right)\operatorname{arctg}\dfrac{y}{x}.$

6.24. $f(x,y) = xe^y + ye^x.$

6.25. $f(x,y) = \operatorname{arcctg}\dfrac{x+y}{1-xy}.$

6.26. $f(x,y) = \arcsin\dfrac{x}{\sqrt{x^2+y^2}}.$

6.27. $f(x,y) = e^{\frac{x}{y}}\ln\dfrac{x}{y}.$

6.28. $f(x,y) = e^{\frac{x}{y}}\ln\dfrac{y}{x}.$

6.29. $f(x,y) = \arccos\dfrac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}}.$

6.30. $f(x,y) = (\cos x)^{\sin y}.$

7. Să se arate că funcţiile următoare verifică relaţiile indicate, în ipoteza că ele sunt diferenţiabile de ordinul cerut de relaţiile respective:

7.1. $f(x,y) = e^x \cos y$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0.$

7.2. $f(x,y) = \dfrac{xy}{x-y}$ verifică $\dfrac{\partial^2 f}{\partial x^2} + 2\dfrac{\partial^2 f}{\partial x \partial y} + \dfrac{\partial^2 f}{\partial y^2} = \dfrac{2}{x-y}.$

7.3. $f(x,y) = \ln(e^x + e^y)$ verifică $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y} = 1.$

7.4. $f(x,y) = \ln(e^x + e^y)$ verifică $\dfrac{\partial^2 f}{\partial x^2} \cdot \dfrac{\partial^2 f}{\partial y^2} = \left(\dfrac{\partial^2 f}{\partial x \partial y}\right)^2.$

7.5. $f(x,y) = \ln\left(x^2 + y^2\right)$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0.$

7.6. $f(x,y) = e^x (x \cos y - y \sin y)$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0.$

7.7. $f(x,y) = \ln\left(x^2 + xy + y^2\right)$ verifică $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = 2.$

7.8. $f(x,y) = \ln\sqrt{(x-a)^2 + (y-b)^2}$, $\{a,b\} \subset \mathbb{R}$ verifică $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0.$

7.9. $f(x,y) = x^y y^x$ verifică $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = (x + y + \ln f(x,y))\, f(x,y).$

7.10. $f(x,y,z)=(x-y)(y-z)(z-x)$ verifică $\dfrac{\partial f}{\partial x}+\dfrac{\partial f}{\partial y}+\dfrac{\partial f}{\partial z}=0.$

7.11. $f(x,y,z)=\dfrac{1}{\sqrt{x^2+y^2+z^2}}$ verifică $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}+\dfrac{\partial^2 f}{\partial z^2}=0.$

7.12. $f(x,y,z)=\dfrac{1}{x-y}+\dfrac{1}{y-z}+\dfrac{1}{z-x}$

verifică $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}+\dfrac{\partial^2 f}{\partial z^2}+2\left(\dfrac{\partial^2 f}{\partial x\partial y}+\dfrac{\partial^2 f}{\partial y\partial z}+\dfrac{\partial^2 f}{\partial z\partial x}\right)=0.$

7.13. $f(x,y,z)=\ln\left(e^x+e^y+e^z\right)$ verifică $\dfrac{\partial f}{\partial x}+\dfrac{\partial f}{\partial y}+\dfrac{\partial f}{\partial z}=1.$

7.14. $f(x,y,z,t)=\dfrac{x-y}{z-t}+\dfrac{t-x}{y-z}$ verifică $\dfrac{\partial f}{\partial x}+\dfrac{\partial f}{\partial y}+\dfrac{\partial f}{\partial z}+\dfrac{\partial f}{\partial t}=0.$

8. Să se calculeze $\dfrac{\partial f}{\partial t}$, unde $f=f(x,y),\ x=\varphi(t),\ y=\psi(t)$:

8.1. $f(x,y)=x^2y^3,\quad x=t,\ y=t^2.$

8.2. $f(x,y)=x^2-xy+y^2,\quad x=\cos t,\ y=\sin t.$

8.3. $f(x,y)=xy^2-x^2y,\quad x=\sin t,\ y=\cos t.$

8.4. $f(x,y)=e^{xy}\ln(x+y),\quad x=1-t^3,\ y=t^3.$

8.5. $f(x,y)=e^{x-2y},\quad x=\sin t,\ y=t^3.$

8.6. $f(x,y)=\ln\left(e^x+e^y\right),\quad x=t^2,\ y=1-t^2.$

8.7. $f(x,y)=x^2+xy+y^2,\quad x=t^3,\ y=t^2.$

8.8. $f(x,y)=e^{2\left(x^2-y^2\right)},\quad x=\cos t,\ y=\sin t.$

8.9. $f(x,y) = \ln\sin\dfrac{x}{\sqrt{y}}, \quad x = 3t^2,\ y = \sqrt{t^2+1}.$

8.10. $f(x,y) = x^y, \quad x = \cos x,\ y = 2x.$

9. Să se calculeze $\dfrac{\partial f}{\partial x}$ şi $\dfrac{\partial f}{\partial y}$, dacă $f = f(u,v)$, $u = \varphi(x,y)$, $v = \psi(x,y)$:

9.1. $f(u,v) = u^2 \ln v, \quad u = \dfrac{y}{x},\ v = x + 2y.$

9.2. $f(u,v) = u^2 - v^2, \quad u = x\sin y,\ v = x\cos y.$

9.3. $f(u,v) = u^2 + \sqrt{uv}, \quad u = x + y,\ v = \dfrac{x}{y}.$

9.4. $f(u,v) = \sqrt[3]{u} + \dfrac{1}{\cos v}, \quad u = xy,\ v = x - y.$

9.5. $f(u,v) = uv \operatorname{arctg} uv, \quad u = t^3,\ v = t^2 + 1.$

9.6. $f(u,v) = u\sin v + v\cos u, \quad u = \dfrac{x}{y},\ v = xy.$

9.7. $f(u,v) = \operatorname{arctg}\dfrac{v}{u}, \quad u = x\cos y,\ v = x\sin y.$

9.8. $f(u,v) = u^v, \quad u = y\sin x,\ v = x\cos y.$

9.9. $f(u,v) = u^2 + v^2, \quad u = \dfrac{2y}{x+y},\ v = x^2 - 3y.$

9.10. $f(u,v) = \ln\left(u^2 + v^2 + 1\right), \quad u = \sin\dfrac{x}{y},\ v = \sqrt{\dfrac{x}{y}}.$

10. Să se calculeze diferenţiala de ordinul I pentru funcţiile următoare:

10.1. $f(x,y) = x^3y^2 + xy^3 + 2.$ 10.2. $f(x,y) = xye^{\frac{x}{y}}.$

10.3. $f(x,y) = x^2 + \sin 3y.$ 10.4. $f(x,y) = \dfrac{x+y}{2x-3y}.$

10.5. $f(x,y) = \ln\left(x+y^2\right).$ 10.6. $f(x,y) = \ln \operatorname{tg} \dfrac{x}{y}.$

10.7. $f(x,y) = x^2y + xy^3 + y^3.$ 10.8. $f(x,y) = \sin x \cos y.$

10.9. $f(x,y) = x\sqrt{y} + \dfrac{y}{\sqrt{x}}.$ 10.10. $f(x,y) = \left(x^2+y^2\right)^5.$

10.11. $f(x,y) = \dfrac{y^2}{x^3}.$ 10.12. $f(x,y) = e^{x^2+y^2}.$

10.13. $f(x,y) = \cos 2x + \sin 2x.$ 10.14. $f(x,y) = y\cos x^2 + x\sin y^2.$

10.15. $f(x,y) = x^2 + y^2 + \sin xy.$ 10.16. $f(x,y) = \sqrt[3]{x^2+y^2}.$

10.17. $f(x,y,z) = xyz.$ 10.18. $f(x,y,z) = x^{y^z}.$

10.19. $f(x,y,z) = \sin(x+y+z).$ 10.20. $f(x,y,z) = \arcsin \dfrac{z}{\sqrt{x^2+y^2+z^2}}.$

11. Să se scrie diferenţialele de ordin II pentru funcţiile:

11.1. $f(x,y) = x^3 - x^2y + 2y^3 + 3x - 2y + 5.$ 11.2. $f(x,y) = e^{xy}.$

11.3. $f(x,y) = 5x^2y + 3xy + y^2 + 3.$ 11.4. $f(x,y) = \dfrac{x}{y}e^{xy}.$

11.5. $f(x,y)=\sqrt{1+2xy+y^2}$. 11.6. $f(x,y)=e^y\sin x$.

11.7. $f(x,y)=\ln\left(x^2+y\right)$. 11.8. $f(x,y)=\left(x^3+y^2\right)^2$.

11.9. $f(x,y)=x^2+y^2+\cos xy$. 11.10. $f(x,y)=\dfrac{1}{\sqrt[3]{x^2+y^2}}$.

11.11. $f(x,y)=\dfrac{x}{y}-\dfrac{y}{x}$. 11.12. $f(x,y)=y\ln\dfrac{x}{y}$.

11.13. $f(x,y)=\operatorname{arcctg}\dfrac{y}{x+y}$. 11.14. $f(x,y)=e^x\operatorname{tg}y$.

11.15. $f(x,y)=\arcsin\dfrac{x}{\sqrt{x^2+y^2}}$. 11.16. $f(x,y)=e^{xy^2}$.

11.17. $f(x,y)=xe^y+ye^x$. 11.18. $f(x,y)=(\sin x)^{\cos y}$.

11.19. $f(x,y)=\sqrt[3]{x^4}+\sqrt{y^3}$. 11.20. $f(x,y)=\left(x^2+y^2\right)\operatorname{arctg}\dfrac{x}{y}$.

12. Utilizând diferenţiala, să se calculeze cu aproximaţie:

12.1. $\sqrt{1,01^3+1,98^3}$. 12.2. $(3,01)^{2,03}$.

12.3. $\sqrt[3]{(5,02)^2+(1,41)^2}$. 12.4. $(2,02)^{3,01}$.

12.5. $\sin 29^\circ\cos 62^\circ$. 12.6. $\sin 31^\circ\operatorname{tg}46^\circ$.

12.7. $\operatorname{arctg}\left(\dfrac{1,98}{1,03}-1\right)$. 12.8. $\operatorname{arcctg}\left(\dfrac{1,97}{1,01}-1\right)$.

12.9. $\ln\left(\sqrt[3]{1,02}+\sqrt[4]{0,98}-1\right)$. 12.10. $\dfrac{1,02^{3,01}}{\sqrt[3]{0,99\sqrt[4]{1,03^5}}}$.

13. Să se scrie formula Taylor (până la termenii de gradul III inclusiv) corespunzătoare următoarelor funcţii în punctele indicate:

13.1. $f(x,y) = xy^3 + 2xy - 2x^2 + 3x + y - 2, \quad (-1,2).$

13.2. $f(x,y) = x^3 - 3xy^2 + y^3 + 2x - 3y + 1, \quad (1,2).$

13.3. $f(x,y) = x^3 - 5x^2 - xy + y^2 + 10x + 5y + 10, \quad (1,-1).$

13.4. $f(x,y) = \sqrt[3]{x+y}, \quad (0,1).$

13.5. $f(x,y) = \ln(1+x+y), \quad (1,0).$

13.6. $f(x,y) = e^x \sin y, \quad \left(0, \dfrac{\pi}{2}\right).$

13.7. $f(x,y) = e^{2y} \ln(1+x), \quad (0,0).$

13.8. $f(x,y) = x^y, \quad (1,1).$

13.9. $f(x,y) = e^y \cos x, \quad (0,\pi).$

13.10. $f(x,y) = \ln(1+x)\ln(1+y), \quad (1,1).$

14. Să se determine valorile maxime şi minime ale următoarelor funcţii $f : \mathbb{R}^2 \longrightarrow \mathbb{R}$:

14.1. $f(x,y) = x^3 + y^3 - 9xy + 18.$

14.2. $f(x,y) = x^4 + y^4 - 4xy + 2.$

14.3. $f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 2.$

14.4. $f(x,y) = -x^2 - xy - y^2 + x + y.$

14.5. $f(x,y) = x^2 + xy + y^2 + \dfrac{1}{x} + \dfrac{1}{y}.$

14.6. $f(x,y) = x^3 + y^3 - 6xy.$

14.7. $f(x,y) = 3x^2 - x^3 + 3y^2 + 4y.$

14.8. $f(x,y) = x^3 + 3xy^2 - 15x - 12y + 8.$

14.9. $f(x,y) = \left(2x^2 + y^2\right) e^{-\left(x^2+y^2\right)}.$

14.10. $f(x,y) = 3 - \sqrt[3]{x^2 + y^2}.$

14.11. $f(x,y) = xy + \dfrac{20}{x} + \dfrac{20}{y}$.

14.12. $f(x,y) = x^2 y e^{y-x}$.

14.13. $f(x,y) = 1 - \sqrt{x^2+y^2}$.

14.14. $f(x,y) = \dfrac{x+y}{\sqrt{x^2+y^2+1}}$.

14.15. $f(x,y) = x + y + 4\sin x \sin y$.

14.16. $f(x,y) = ye^{x+y\sin x}$.

14.17. $f(x,y) = x^3 + y^2 - 3x + 4\sqrt{y^5}$.

14.18. $f(x,y) = x\sqrt{y} - x^2 - y + 6x + 1$.

14.19. $f(x,y) = \left(x+y^2\right)\sqrt{e^x}$.

14.20. $f(x,y) = (x-y)^2 + (x-1)^3$.

15. Să se determine extremele condiţionate ale funcţiilor $f(x,y)$ cu legătura $F(x,y) = 0$:

15.1. $f(x,y) = xy \;, \quad F(x,y) = x^2 + y^2 - 1$.

15.2. $f(x,y) = \cos 2x + \cos 2y \;, \quad F(x,y) = x - y - \dfrac{\pi}{4}$.

15.3. $f(x,y) = xy \;, \quad F(x,y) = x + y - 1$.

15.4. $f(x,y) = x + 2y \;, \quad F(x,y) = x^2 + y^2 - 5$.

15.5. $f(x,y) = x^2 + y^2 - xy + x + y - 4 \;, \quad F(x,y) = x + y + 3$.

15.6. $f(x,y) = xy \;, \quad F(x,y) = x^3 + y^3 - xy$.

15.7. $f(x,y) = e^{xy} \;, \quad F(x,y) = x + y - 1$.

15.8. $f(x,y) = x - y - 4 \;, \quad F(x,y) = x^2 + y^2 - 1$.

15.9. $f(x,y) = x^2 y \;, \quad F(x,y) = 2x + y - 1$.

15.10. $f(x,y) = \dfrac{x}{2} + \dfrac{y}{3} \;, \quad F(x,y) = x^2 + y^2 - 1$.

16. Să se determine extremele globale ale funcţiilor pe domeniile D:

16.1. $f(x,y) = x^2 - y^2 + 2$, $\quad D(x,y) = \left\{(x,y)\,\middle|\,x^2 + y^2 \leqslant 1\right\}$.

16.2. $f(x,y) = x^3 + y^3 - 9xy + 27$, $\quad D(x,y) = \{(x,y)\,|0 \leqslant x \leqslant 4,\ 0 \leqslant y \leqslant 4\}$.

16.3. $f(x,y) = x^3 + y^3 - 3xy$, $\quad D(x,y) = \{(x,y)\,|0 \leqslant x \leqslant 2,\ -1 \leqslant y \leqslant 2\}$.

16.4. $f(x,y) = 2x - y + 3$, $\quad D(x,y) = \{(x,y)\,|x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 2\}$.

16.5. $f(x,y) = x - 2y + 5$, $\quad D(x,y) = \{(x,y)\,|x \leqslant 0,\ y \geqslant 0,\ y - x \leqslant 1\}$.

16.6. $f(x,y) = x^2 + y^2 - xy - x - y$, $\quad D(x,y) = \{(x,y)\,|x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 3\}$.

16.7. $f(x,y) = 2xy$, $\quad D(x,y) = \left\{(x,y)\,\middle|\,x^2 + y^2 \leqslant 4\right\}$.

16.8. $f(x,y) = x^2 y$, $\quad D(x,y) = \left\{(x,y)\,\middle|\,x^2 + y^2 \leqslant 1\right\}$.

16.9. $f(x,y) = x^3 + 4x^2 + y^2 - 2xy$,

D – domeniul închis, mărginit de curbele $y = x^2,\ y = 4$.

16.10. $f(x,y) = xy(4 - x - y)$, $\quad D(x,y) = \{(x,y)\,|x \geqslant 0,\ y \geqslant 0,\ x + y \leqslant 6\}$.