

## Capitolul 4. Integrala nedefinită.

1. Să se calculeze:

$$1.1. \int (x^3 + x^2 + x - 2) dx.$$

$$1.2. \int (x^5 + x^4 + x^{-2} + x^{-3} + 1) dx.$$

$$1.3. \int \left( \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} \right) dx.$$

$$1.4. \int \frac{3 - x - x^2}{x^3} dx.$$

$$1.5. \int \frac{(x^3 + 1)^3}{x^3} dx.$$

$$1.6. \int \frac{(x^2 + 1)^3}{x^4} dx.$$

$$1.7. \int (x^{-\frac{1}{3}} + x^{\frac{2}{3}}) dx.$$

$$1.8. \int (\sqrt{x} + \sqrt[3]{x}) dx.$$

$$1.9. \int \left( \sqrt[3]{x^2} - \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x^2}} + \sqrt[4]{x^5} \right) dx.$$

$$1.10. \int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^3 dx.$$

$$1.11. \int \frac{x^2}{x^2 + 1} dx.$$

$$1.12. \int \frac{x^4}{x^2 + 1} dx.$$

$$1.13. \int \frac{1 - \cos^3 x}{\cos^2 x} dx.$$

$$1.14. \int \frac{1 + 3x^2}{x^2(1 + 2x^2)} dx.$$

$$1.15. \int \frac{2\operatorname{tg}^2 x + 3}{\sin^2 x} dx.$$

$$1.16. \int \frac{x^6 - x^2 + 1}{x^2 + 1} dx.$$

$$1.17. \int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx.$$

$$1.18. \int \frac{dx}{\sin^2 x \cos^2 x}.$$

$$1.19. \int \sin^2 \frac{x}{2} dx.$$

$$1.20. \int \cos^2 \frac{x}{2} dx.$$

$$1.21. \int \left( \cos \frac{x}{2} - \sin \frac{x}{2} \right)^2 dx.$$

$$1.22. \int \left( \cos^4 \frac{x}{2} - \sin^4 \frac{x}{2} \right) dx.$$

$$1.23. \int \operatorname{tg}^2 x dx.$$

$$1.24. \int \operatorname{ctg}^2 x dx.$$

$$1.25. \int \left( 3^{\frac{x}{2}} + 3^{-\frac{x}{2}} \right)^2 dx.$$

$$1.26. \int e^x \left( 1 + \frac{e^{-x}}{\sin^2 x} \right) dx.$$

$$1.27. \int e^x 2^x dx.$$

$$1.28. \int 3^x \left( 2 + \frac{3^{-x}}{\sqrt{1-x^2}} \right) dx.$$

$$1.29. \int \frac{3^x + 4^x}{12^x} dx.$$

$$1.30. \int \left( \frac{2}{\sqrt{1-x^2}} - \frac{3}{1+x^2} \right) dx.$$

**2. Să se calculeze:**

$$2.1. \int (2x+5)^3 dx.$$

$$2.2. \int (3x-7)^5 dx.$$

$$2.3. \int (4-x)^{10} dx.$$

$$2.4. \int \sqrt[3]{(2x-7)^5} dx.$$

$$2.5. \int \sqrt[4]{\left(1 - \frac{x}{2}\right)^3} dx.$$

$$2.6. \int \sqrt{3x-7} dx.$$

$$2.7. \int \sin 5x dx.$$

$$2.8. \int \sin(3x-2) dx.$$

$$2.9. \int \cos 4x dx.$$

$$2.10. \int \cos\left(3 - \frac{x}{2}\right) dx.$$

$$2.11. \int \frac{dx}{\cos^2 7x}.$$

$$2.12. \int \frac{dx}{\cos^2\left(\frac{x}{2} - 3\right)}.$$

$$2.13. \int \frac{dx}{\sin^2 \frac{x}{3}}.$$

$$2.14. \int \frac{dx}{\sin^2(3-2x)}.$$

$$2.15. \int \frac{dx}{\sqrt{1-4x^2}}.$$

$$2.16. \int \frac{dx}{\sqrt{4-x^2}}.$$

$$2.17. \int \frac{dx}{1+9x^2}.$$

$$2.18. \int \frac{dx}{1+\frac{x^2}{4}}.$$

$$2.19. \int \frac{dx}{x+1}.$$

$$2.20. \int \frac{dx}{3x-1}.$$

$$2.21. \int e^{-x} dx.$$

$$2.22. \int e^{3x-1} dx.$$

$$2.23. \int 2^{3x+1} dx.$$

$$2.24. \int 5^{1-2x} dx.$$

**3. Să se calculeze:**

$$3.1. \int (x^2 + 3x + 1)^{10} (2x + 3) dx. \quad 3.2. \int \sqrt{3 - 2x + x^2} (x - 1) dx.$$

$$3.3. \int \sqrt[3]{x^3 - 8} x^2 dx. \quad 3.4. \int \sqrt{x^4 + 1} x^3 dx.$$

$$3.5. \int \frac{2x - 3}{x^2 - 3x + 7} dx. \quad 3.6. \int \frac{6x - 7}{3x^2 - 7x + 10} dx.$$

$$3.7. \int \frac{e^{\sqrt{x+1}}}{\sqrt{x+1}} dx. \quad 3.8. \int e^{\sin x} \cos x dx.$$

$$3.9. \int \frac{e^{\arcsin x}}{\sqrt{1-x^2}} dx. \quad 3.10. \int \frac{e^{\operatorname{arctg} x} + 1}{1+x^2} dx.$$

$$3.11. \int e^{x^3} x^2 dx. \quad 3.12. \int \sin^3 x \cos x dx.$$

$$3.13. \int \cos^5 x \sin x dx. \quad 3.14. \int \sin^7 x \cos x dx.$$

$$3.15. \int \frac{\sin x}{\cos^5 x} dx. \quad 3.16. \int \frac{\cos x}{1 + 2 \sin x} dx.$$

$$3.17. \int \frac{dx}{x \ln x}. \quad 3.18. \int \frac{dx}{x (\ln x + 2)}.$$

$$3.19. \int \frac{\sqrt[3]{(1 + \ln x)^2}}{x} dx. \quad 3.20. \int \frac{(\ln x + 4)^2}{x} dx.$$

$$3.21. \int \frac{\operatorname{arctg}^2 x}{1+x^2} dx. \quad 3.22. \int \frac{\operatorname{arctg} 2x + x}{1+4x^2} dx.$$

$$3.23. \int \frac{\arcsin^2 x - 1}{\sqrt{1-x^2}} dx. \quad 3.24. \int \sqrt{\frac{\arccos x}{x^2 - 1}} dx.$$

$$3.25. \int \frac{dx}{\sin x}.$$

$$3.26. \int \frac{dx}{\cos x}.$$

$$3.27. \int \frac{2^x}{\sqrt{1-4^x}} dx.$$

$$3.28. \int \frac{6x-7}{3x^2-7x+10} dx.$$

$$3.29. \int 3^{2x^2+x-1} (4x+1) dx. \quad 3.30. \int x e^{-x^2} dx.$$

#### 4. Să se calculeze:

$$4.1. \int \frac{1}{x^2} \sin \frac{1}{x} dx.$$

$$4.2. \int \frac{dx}{\sqrt{x} + \sqrt[4]{x}}.$$

$$4.3. \int \frac{dx}{\sin^2 x + 4 \cos^2 x}.$$

$$4.4. \int \frac{dx}{1 + \sin^2 x}.$$

$$4.5. \int \frac{6 \operatorname{tg} x}{3 \sin 2x + 5 \cos^2 x} dx.$$

$$4.6. \int \frac{8 + \operatorname{tg} x}{18 \sin^2 x + 2 \cos^2 x} dx.$$

$$4.7. \int \frac{\ln \arcsin x}{\sqrt{1-x^2} \arcsin x} dx.$$

$$4.8. \int \ln \frac{1+x}{1-x} \cdot \frac{1}{x^2-1} dx.$$

$$4.9. \int \frac{e^{\operatorname{ctg} 2x} + \operatorname{tg} 2x}{\sin^2 2x} dx.$$

$$4.10. \int \frac{\operatorname{arctg} \sqrt{x}}{(1+x) \sqrt{x}} dx.$$

$$4.11. \int \frac{\cos^3 x}{\sqrt{\sin x}} dx.$$

$$4.12. \int \frac{dx}{x^2 \sqrt{1+x^2}}.$$

$$4.13. \int \frac{dx}{\sqrt{-8-6x-x^2}}.$$

$$4.14. \int \frac{dx}{x^2+4x+5}.$$

$$4.15. \int \frac{x+1}{x^2-x+1} dx.$$

$$4.16. \int \frac{3x^2-2x}{x^3-x^2+1} dx.$$

$$4.17. \int \sqrt{\frac{1+x}{1-x}} \cdot \frac{1}{1-x} dx.$$

$$4.18. \int \frac{e^x-1}{e^x+1} dx.$$

$$4.19. \int \frac{7x+3}{x^2+4} dx.$$

$$4.20. \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}.$$

$$4.21. \int \frac{3 \cos x + 2 \sin x}{(3 \sin x - 2 \cos x)^2} dx.$$

$$4.22. \int \frac{3 \cos x - 2 \sin x}{(2 \cos x + 3 \sin x)^3} dx.$$

**5. Să se calculeze:**

5.1.  $\int x \sin x \, dx.$

5.2.  $\int x \cos x \, dx.$

5.3.  $\int x^2 \sin x \, dx.$

5.4.  $\int (x^2 - x + 1) \cos x \, dx.$

5.5.  $\int x e^x \, dx.$

5.6.  $\int x^2 e^x \, dx.$

5.7.  $\int x e^{3x} \, dx.$

5.8.  $\int (x^2 + x - 1) e^x \, dx.$

5.9.  $\int x \ln x \, dx.$

5.10.  $\int x^2 \ln x \, dx.$

5.11.  $\int (x^2 + x + 1) \ln x \, dx.$

5.12.  $\int \ln x \, dx.$

5.13.  $\int x^n \ln x \, dx, \, n \in \mathbb{N}.$

5.14.  $\int \ln^2 x \, dx.$

5.15.  $\int x \operatorname{arctg} x \, dx.$

5.16.  $\int \operatorname{arctg} x \, dx.$

5.17.  $\int \arccos^2 x \, dx.$

5.18.  $\int \arcsin^2 x \, dx.$

5.19.  $\int x^2 3^x \, dx.$

5.20.  $\int x 2^x \, dx.$

5.21.  $\int e^x \sin x \, dx.$

5.22.  $\int e^x \cos x \, dx.$

5.23.  $\int \cos(\ln x) \, dx.$

5.24.  $\int \sin(\ln x) \, dx.$

5.25.  $\int \frac{x}{\sin^2 x} \, dx.$

5.26.  $\int \frac{x}{\cos^2 x} \, dx.$

5.27.  $\int \frac{\operatorname{arctg} x}{x^2} \, dx.$

5.28.  $\int \frac{\arcsin x}{\sqrt{1+x}} \, dx.$

**6. Să se calculeze:**

6.1.  $\int \frac{x-5}{(x-3)(x-4)} dx.$

6.2.  $\int \frac{2x+5}{(x-1)(x+6)} dx.$

6.3.  $\int \frac{dx}{(x+2)(x-1)}.$

6.4.  $\int \frac{x+1}{(x+2)(x+3)} dx.$

6.5.  $\int \frac{dx}{(x-3)(x-2)(x+1)}.$

6.6.  $\int \frac{x^3+3x^2+3x+1}{x(x+2)(x+3)} dx.$

6.7.  $\int \frac{x^3-3x^2+3x}{(x-1)(x-2)} dx.$

6.8.  $\int \frac{2x^4-5x^2-8x-8}{x(x-2)(x+2)} dx.$

6.9.  $\int \frac{dx}{(x-2)(x+1)(x+2)}.$

6.10.  $\int \frac{x^2-x-9}{x^2-x-6} dx.$

6.11.  $\int \frac{x^2+2x+2}{x(x+2)(x-1)(x+3)} dx.$

6.12.  $\int \frac{x^2+2x-11}{(x-1)(x+3)(x-5)} dx.$

6.13.  $\int \frac{dx}{x^2(x+2)}.$

6.14.  $\int \frac{x^2+4x+6}{(x+2)^2x} dx.$

6.15.  $\int \frac{x^5-2x^2+3}{(x-2)^2} dx.$

6.16.  $\int \frac{2x+1}{(x-1)^3} dx.$

6.17.  $\int \frac{x^2-2x+3}{x^2(x-2)} dx.$

6.18.  $\int \frac{x+1}{(x-1)^2(x-3)} dx.$

6.19.  $\int \frac{5x-1}{(x-1)^2(x-2)} dx.$

6.20.  $\int \frac{dx}{x^2(x+5)^2}.$

6.21.  $\int \frac{dx}{x(x+1)^2(x+2)^3}.$

6.22.  $\int \frac{x}{(x+1)^2(x+2)^2(x-1)} dx.$

**7. Să se calculeze:**

7.1.  $\int \frac{dx}{x^3 + 8}.$

7.2.  $\int \frac{dx}{x(x^2 + 1)}.$

7.3.  $\int \frac{dx}{(x+1)(x^2+2)}.$

7.4.  $\int \frac{x-2}{x(x^2+4)} dx.$

7.5.  $\int \frac{x}{x^3+1} dx.$

7.6.  $\int \frac{dx}{(x-2)(x-4)(x^2+2x+2)}.$

7.7.  $\int \frac{x^4}{x^4-1} dx.$

7.8.  $\int \frac{x-1}{x(x^2+1)} dx.$

7.9.  $\int \frac{x^3+4x^2+3x+2}{(x+1)^2(x^2+1)} dx.$

7.10.  $\int \frac{x(x^2+2x+10)}{(x+1)^2(x^2-x+1)} dx.$

7.11.  $\int \frac{dx}{x^2(x^2-2x+2)}.$

7.12.  $\int \frac{3x^2-6x+1}{(x+1)^2(3x^2-8x+9)} dx.$

7.13.  $\int \frac{x-1}{(x^2+1)^2} dx.$

7.14.  $\int \frac{x^4+2x^2+4}{(x^2+1)^3} dx.$

7.15.  $\int \frac{x^2+2x+7}{(x-2)(x^2+1)^2} dx.$

7.16.  $\int \frac{3x+1}{x(1+x^2)^2} dx.$

7.17.  $\int \frac{5x+8}{(x^2+4)^2} dx.$

7.18.  $\int \frac{3x+5}{(x^2+2x+2)^2} dx.$

7.19.  $\int \frac{x^2}{(x+1)^2(x^2-x+1)} dx.$

7.20.  $\int \frac{2x^4+5x^2-2}{2x^3-x-1} dx.$

**8. Să se calculeze:**

8.1.  $\int \frac{dx}{3 \sin x + 4 \cos x}.$

8.2.  $\int \frac{dx}{\sin 2x + \cos^2 x}.$

8.3.  $\int \frac{dx}{\sin x - \cos x}.$

8.4.  $\int \frac{dx}{3 \sin x + 4 \cos x + 5}.$

8.5.  $\int \frac{3 \sin x + 2 \cos x}{\sin^2 x \cos x + 4 \cos^3 x} dx.$

8.6.  $\int \frac{\sin x + 3 \cos x}{\sin^2 x \cos x + \cos^3 x} dx.$

8.7.  $\int \frac{\sin x(1 + \sin^2 x)}{\cos 2x} dx.$

8.8.  $\int \frac{\cos^3 x(1 + \cos^2 x)}{\sin^2 x(1 + \sin^2 x)} dx.$

8.9.  $\int \cos 2x \cos 4x dx.$

8.10.  $\int \cos 3x \cos x \cos 5x dx.$

8.11.  $\int \sin x \sin 3x dx.$

8.12.  $\int \sin 2x \sin 4x \sin 6x dx.$

8.13.  $\int \sin x \cos 3x dx.$

8.14.  $\int \sin 2x \cos 4x \cos 6x dx.$

8.15.  $\int \sin^2 2x \cos^2 2x dx.$

8.16.  $\int \sin^4 x \cos^4 x dx.$

8.17.  $\int \sin^2 x \cos^4 x dx.$

8.18.  $\int \sin^4 x \cos^2 x dx.$

8.19.  $\int \sin^2 x \cos^3 x dx.$

8.20.  $\int \sin^3 x \cos^2 x dx.$

8.21.  $\int \cos^5 x dx.$

8.22.  $\int \sin^3 x \cos^3 x dx.$