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# Mathematical modeling of high-speed loads effects on underground storage tanks*

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**Abstract**

The purpose of this paper is to provide a numerical modeling of intense dynamic loads on engineering materials. Numerical results illustrate the evolution of the state of elastic-plastic shells (filled with fluid or elastic-plastic material), which are subjected to explosive loads.

**Keywords:** numerical modeling, elastic-plastic media, high-speed loads, numerical method.

## 1 Introduction

The elements of thin-wall constructions are used in various fields of engineering practice and most of them function in conditions of rather high operational load [1-3]. These objects include different shell containers constructed of materials with diverse physical and mechanical properties and intended to store flammable, toxic and chemical substances. Their stress-strain state exposed to a wide range of external forces and force-majeure circumstances represents a significant interest for analysis in order to level down the accidental risk and environmental impact. In this paper the behavior of shell containers deepened in ground and exposed to intense dynamic loads is considered [4, 5]. To adequately describe all the processes arising under intense dynamic loads of the material it is important to choose a particular mathematical model of the environment. This model should take into account a noninvertible nature of deformation, the dependence on strain rates



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and other processes associated with an explosive loading. The behavior of various materials is described within the framework of state equation in the form of Mie-Gruneisen, taking into account the complex stress-strain behavior of the matter. The ground is represented as a three-component substance consisting of solid granules, water and air [3]. Computer modeling of deformation under explosive loading supposes setting a numerical method and obtaining a picture of internal parameters' evolution. In this paper following Wilkins [6-8] we consider a second order accurate finite-difference scheme and provide its modification to solving specific problems. The use of adaptive meshes, parallel numerical algorithms and multiprocessor clusters allows one to reduce computing time [9].

## 2 Setting of the problem and mathematical model

Assume an underground container is subjected to explosive loading. Note that explosives are situated inside and/or outside the shell. The behavior of theses structures under intense dynamic loads is of unsteady nature and is described with the help of environment models and physical laws of loading. Let us examine the dynamics of explosive loading within the framework of two-dimensional model of elastic-plastic medium and solve the following basic equations [6]:

$$
\begin{gathered}
\sigma' = k_0 \left( \varepsilon_{kk} - \alpha_\nu \left(T - T_0\right) - \frac{\Lambda}{3} \int_0^\omega \frac{\partial \dot{\omega}}{\partial \sigma} \partial \omega \right) \\
\left(\acute{\tau_{ij}}\right)^\nabla + \lambda \acute{\tau_{ij}} = 2\mu_0 \dot{e_{ij}}, \acute{\tau_{ij}} \acute{\tau_{ij}} \le \frac{2}{3} Y_0^2, \\
\rho c_0 \dot{T} + \alpha_\nu \dot{\sigma} T = \tau_{ij} \dot{\varepsilon_{ij}}^p + \Lambda \dot{\omega}^2 - div \bar{q}, \\
\dot{\omega} = B(\sigma' - \sigma_*)^m H(\sigma' - \sigma_*), \\
\tau_{ij} = S_{ij} + \Gamma \varepsilon_{ij}, \quad \acute{\tau_{ij}} = \tau_{ij}/(1-\omega), \sigma' = \sigma/(1-\omega).
\end{gathered} \tag{1}
$$

Here the symbol $T$ denotes temperature; $\rho$ – density; $\bar{q}$ – heat transfer rate; $\sigma_{ij} = \sigma\delta_{ij} + s_{ij}$ – stress component, divided into two mutually orthogonal tensors, spherical tensor $\sigma\delta_{ij} = \sigma_{kk}\delta_{ij}/3$ and deviator $s_{ij}$; $\varepsilon_{ij}, \varepsilon^e_{ij}, \varepsilon^p_{ij}$ – elastic and plastic strain components; $e_{ii} = \varepsilon_{ij} - 1/3\varepsilon_{kk}\delta_{ij}$ – strain deviator components; $\omega$ – structural parameter describing the origin and growth of material's vulnerability; $H(x)$ – Heviside function; $\delta_{ii}$ – Kronecker delta; $k_0$ and $\mu_0$ – bulk modulus and shear modulus of undamaged material; $\alpha_\nu$ – cubic expansion coefficient; $c_\sigma$ – heat at constant pressure; $\tau_{ij}$ – components of the "active" stress tensor; $A, B, m, \Gamma$ – material's characteristics. We assume $\varepsilon_{ij} = \varepsilon^e_{ij} + \varepsilon^p_{ij}$ and $\varepsilon^p_{kk} = 0$, then plastic flow is incompressible. $\nabla$ is a Yauman's derivative of tensor components:

$$S^{\nabla}_{ij} = \dot{S_{ij}} - S_{ik}\omega_{jk} - S_{jk}\omega_{ik}; \quad (2)$$

$$\acute{\omega_{ij}} = 1/2(\frac{\partial \nu_i}{\partial x_j} - \frac{\partial \nu_j}{\partial x_i}),$$

$\nu$ – velocity components, $x$ – Cartesian coordinates; $\lambda$ is determined by the Mises plastic's:

$\lambda = 0$ in elastic region,

$\lambda = 3\mu_0\acute{\tau_{ij}}\dot{e}_{ij}H(\acute{\tau_{ij}}\dot{e}_{ij})/Y^2$ in the region of plastic flow.

The model used in this work generalizes the Prandtl-Reiss elastoplastic flow model along with the Mises plasticity criterion, as well as accounts for the anisotropy of plastic deformation (for $\Gamma \neq 0$ ), the formation of microdamages in rarefaction waves, and thermal effects [5]. It is supposed that the flow limit $Y$, modules $k_0$ and $\mu_0$ depend on the temperature, pressure and other state parameters (Steinberg-Guinan model) [7]:

$$Y = Y_0(1 + \beta\varepsilon^p_u)^n(1 - b\sigma(\frac{\rho_0}{\rho})^{1/3} - h(T - T_0)),$$

$$Y_0(1 + \beta\varepsilon^p_u)^n \leq Y_{\max}, \quad Y_0 = 0 \quad at \quad T > T_m, \quad (3)$$

$$T_m = T_{m0}(\frac{\rho_0}{\rho})^{2/3}\exp(2\gamma_0(1 - \frac{\rho_0}{\rho})),$$

$$\mu_0 = \mu_{00}(1 - b\sigma(\frac{\rho_0}{\rho})^{1/3} - h(T - T_0)),$$

where $\varepsilon_u^p = \sqrt{2\varepsilon_{ij}^p \varepsilon_{ij}^p / 3}$ – is the plastic deformation tensor intensity; $\mathrm{T}_m$ – melting temperature of the material; $Y_0, Y_{\max}, T_{m0}, \beta, b, \gamma_0, \mu_{00}$ – material constants. It is considered that $\sigma_* = \sigma_*^0 Y / Y^0, \sigma_*^0$ is a material constant. Numerical modeling of impulsive impacts gives a possibility to neglect strain anisotropy of the materials (in case of adiabatic flow) and to assume $divq = 0$ and $\Gamma = 0$ in equation (1). Thus from equation (1) it follows that:

$$s_{ij}^{\nabla} + \lambda s_{ij} = 2\mu \dot{e}_{ij}, \quad s_{ij} s_{ij} \leq \frac{2}{3} Y^2. \tag{4}$$

Equations (4) written for the deviator components of strain tensor are amplified with state equation for the spherical part of strain tensor $\sigma = -p$ ($p$ – pressure):

$$p = p(\rho, U), \tag{5}$$

here $U$ – specific internal energy. State equation is considered in Mie-Gruneisen type [2,7] written in the form:

$$p = l_1 (1 - \frac{\rho_0}{\rho}) + l_2 (1 - \frac{\rho_0}{\rho})^2 + l_3 (1 - \frac{\rho_0}{\rho})^3 + \gamma_0 \rho_0 U, \tag{6}$$

where $l_1, l_2, l_3, \gamma_0$ are material constants that are known for the wide class of materials used in calculations. The whole variety of grounds with different mechanical and physical properties is presented as porous multicomponent medium consisting of solid particles, liquid (water) and gas (air). Characteristics of different types of grounds depend on structure, shape and location of solid particles, percentage of gas and liquid. Following Lyakhov, a ground is called non water-saturated (or air-dry grounds) [3] if air content by volume is much higher than the water content in it. Otherwise the ground is called water-saturated. Thus the ground may be considered as three-component medium (solid particles, water and air) and its characteristics depend on the volume content of each component which can vary over a wide range. Let $d_i$ denote respectively a volume content of air ($i = 1$), water ($i = 2$), and solid ($i = 3$) components in the ground. These values are related: $d_1 + d_2 + d_3 = 1$. If $\rho_0$ is an initial density and $p_0$ is an initial pressure then

$$\rho_0 = \sum_{i=1}^{3} d_i \rho_i.$$

On this assumption, following Lyakhov, we have ground's state equation [3]:

$$\frac{\rho_0}{\rho} = \sum_{i=1}^{3} d_i [\frac{k_i(p - p_0)}{\rho_i c_i^2} + 1]^{-\frac{1}{k_i}}. \quad (7)$$

Here $k_i$ – isentropic exponents, $c_i$ – sound velocities in these components for the case of initial pressure (atmospheric pressure).

In actual numerical calculations instead of formula (7) it is more convenient to use the dependence of pressure on density in an explicit form. This dependence can be approximated by a cubic polynomial with respect to compression $\mu = \frac{\rho_0}{\rho} - 1$ :

$$p = a_0 + a_1\mu + a_2\mu^2 + a_3\mu^3. \quad (8)$$

Polynomial coefficients are determined on account on (7) using Newton interpolation polynomial or the method of least squares.

## 3 Numerical results

Initial values of other parameters are as follows: air – $k_1 = 1,4$, $\rho_1 = 12 * 10^4$ gr/sm$^3$, $c_1 = 300$ m/sec; water – $k_2 = 3,0$, $\rho_2 = 1,0$ gr/sm$^3$, $c_2 = 1500$ m/sec; quartz – $k_3 = 3,0$, $\rho_3 = 2,65$ gr/sm$^3$, $c_3 = 4500$ m/sec. At small values of pressure ground's compressibility depends on compressibility of air which significantly exceeds the compressibility of water and quartz. At higher values of pressure ground's compressibility depends on compressibility of liquid and solid components. The discrepancy between the data obtained from expressions (7) and (8) is equal to less than 10 percent.

Consider a water-saturated ground with the following composition: 10 percent of air, 32 percent of water and 58 percent of quartz. Various numerical experiments were conducted to study the behavior of shells in the ground which are exposed to intense dynamic loading. These

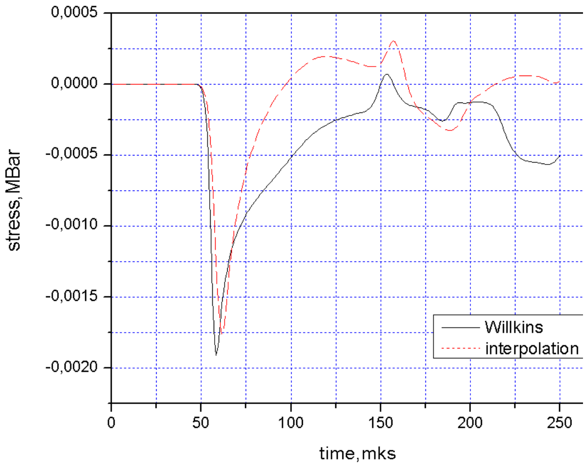


Figure 1. The strain corresponding to different state equations.

studies include an approximation of equations (1)-(6) using a finite-difference scheme of the second order of accuracy that is a development of Wilkins' scheme [2]. Relation (8) is taken as a ground's state equation. It approximates the equation of state with a cubic polynomial. Equation of state (8) is tested for the various cases of a-priory known state equations of medium. Let's consider the case of water and calculate the stress dynamics at a certain point on the computational domain using state equation in Mie-Gruneisen form (solid line in Fig.1), and equation (8) (dotted line in Fig. 1). The expansion of detonation products, shell loading and other physical processes are modeled as well. Some results are presented in the Figures 2–5. The computational domain is a two-dimensional rectangle $21 \times 4$ cm. A water-filled steel shell, 0,2 cm thick, is situated in this domain (double fat black lines in the Fig. 2 and Fig. 4). Its dimensions are $16 \times 7$ cm. The shell is surrounded with a ground consisting of 10 percent air, 30 percent water and 60 percent quartz. The explosive charge TNT ($0,3 \times 0,1$ cm) closely fits the outside of the envelope (deflected contour in the Fig. 2 and Fig. 4). Certain points are selected to monitor the behavior of the elastoplastic medium parameters. These points are called indicators

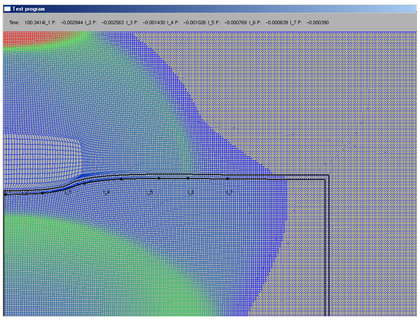

Figure 2. Stress state, t=100 mks

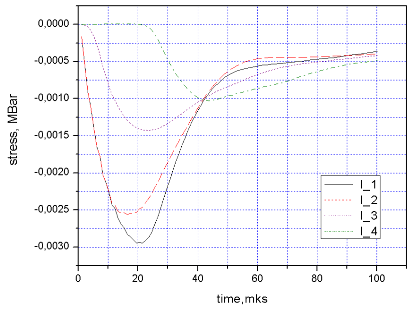


Figure 3. Stress distribution.

and are marked as follows: I-1, I-2, . . . , I-7. They are situated in the inside of the shell.

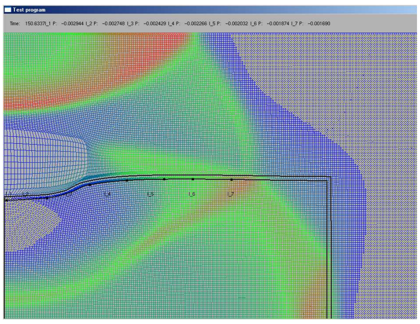

Figure 4. Stress state, t=150 mks

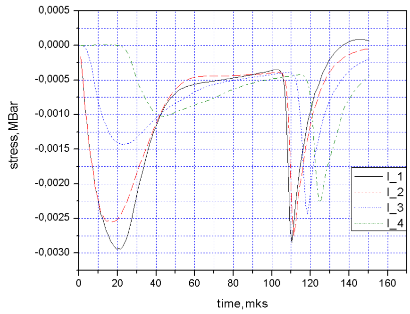


Figure 5. Stress distribution.

Qualitative change in the physical state of the shell and the medium inside and outside it over the time is displayed in Fig. 2 and Fig. 4 at the $t = 100$ mks and $t = 150$ mks correspondingly. Figure 2 displays a wave pattern and the shell's state at $t = 100$ mks in two dimensions. Stress dynamics for the early stages of the loading process up to the $t = 100$ мks is displayed in the Fig. 3. Indicators I-1, I-2 are located directly under the explosive charge (Fig. 2). Thus experimental results for these indicators (Fig. 3) are close enough and have more pronounced wave profile compared to the indicators I-3, I-4 located farther. The stress state of the shell at the $t = 150$ mks is displayed in the Fig. 4. It is easy to observe that the shell exposed to an explosive loading is deformed and the wave reflected from hard walls of the computational domain. Stress distribution for the later stages of the loading process up to time $t = 150$ mks is displayed in the Fig. 5. One may observe a new surge at the $t = 110$ mks caused by the reflected wave.

## 4 Conclusions

A new mathematical model was developed. A stress-strain state of underground storage tanks, exposed to intense dynamic loads and filled in with water or other materials was simulated. The ground was considered as a three-component medium (solid particles, water and air) and its characteristics depended on the volume content of each component which can vary over a wide range. State equation for the ground is approximated by a cubic polynomial with respect to the degree of compression. Numerical results illustrate the evolution of the coupled problem, namely the interaction of ground and elastoplastic shell under explosive loading. This work is supported by STCU (grant 4624) project.

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