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One class of simply periodic motions of a heavy solid with a fixed point

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Abstract

Some simply periodical motions of a heavy solid body with $A = B = 3C$ inertia principal moments' ratio near the fixed point in the uniform gravitational field are studied, employing the method of point mappings.

In this paper we apply the method of point mappings to the research of simply periodic motions of a heavy body near a fixed point. The Euler-Poisson differential equations describing the rigid body motions have the form [1]:

$$A\dot{p} + (C - B)qr = Mg(z_c\gamma_2 - y_c\gamma_3),$$

$$\dot{\gamma_1} = r\gamma_2 - q\gamma_3, \tag{1}$$

$$(A,\ B,\ C,\ p,\ q,\ r,\ x,\ y,\ z,\ 1,\ 2,\ 3),$$

where:

A, B, C is the principal moment of inertia;

x_c , y_c , z_c are the coordinates of the center of mass;

p, q, r are the components of angular velocity;

$\gamma_1, \gamma_2, \gamma_3$ are vertical guiding cosines;

Mg is the body weight.

The motion equations (l) form the six-order system to solve for the variables γ_1,γ_2,γ_3, p, q, r, depending on Euler angles: ψ is the precession angle, θ is the nutation angle and φ is the angle of the proper rotation.

In the general case they assume the energy integral with constant h, the area integral with constant f and geometrical relation.

The complete system of first integrals of Euler-Poisson equations was found only for three cases. Kovalevskaya case is one of them, and our case we consider to be its perturbation. It refers to the body having unity weight with inertia ellipsoid for which $A = B = 3C = 1$, and the center of mass is situated on its abscissa axis at the unity distance from the fixed point.

Taking into account that the precession angle ψ is an ignorable coordinate, it might be excluded by Routh's method reducing to two the order of the system of motion equations, and we may pass to coordinates x and y on the inertia ellipsoid [2]. According to work [3], we bring system (l) at $A = B$ to the reduced dynamic system with two degrees of freedom:

$$x'' = -\Omega y' + \frac{\partial U}{\partial x}, \quad y'' = \Omega x' + \frac{\partial U}{\partial y}. \tag{2}$$

System (2) assumes Jacobi integral

$$x'^2 + y'^2 = 2U. \tag{3}$$

System (2) structure coincides with the structure of equations system for the motion of the restricted three-bodies problem [4], and, using Jacobi integral, its solutions could be presented in three-dimensional phase space *(x, y, x')*.

The problem of search of the system periodical trajectories reduces to determination of the mapping T invariant points' location. The system periodical trajectory, closed after $2n$ crossings with abscissa axis, will be presented in the surface of section by the set of n consequent points. In the simplest case at $n = 1$the trajectory is called simply periodical [4].

If one periodical trajectory is revealed, then in conformity with the theorem of a continuous dependence ordinary differential equations'system solutions on initial conditions, for f and h, not greatly different from the original parameters of the system, it is possible to find out such x_0, that corresponds to the periodical motion, neighbouring to the given one.

The search of the reduced system periodical trajectories was accomplished with the help of a personal computer by the use of a program, set up for the study of dynamic system motions by the method of point mappings, for initial x_0 and velocity $x'_0 = 0, \quad y' > \quad 0.$

Note, that the motion equation (1), as the integral (3), are invariant under substitution

$$x \to x,\ t \to t,\ y \to \text{-}y,\ f \to -f.$$

That's why we can choose the plane (x, x') as surface of section [4]. Such a choice is justified by the fact that φ and x differ by a constant factor only.

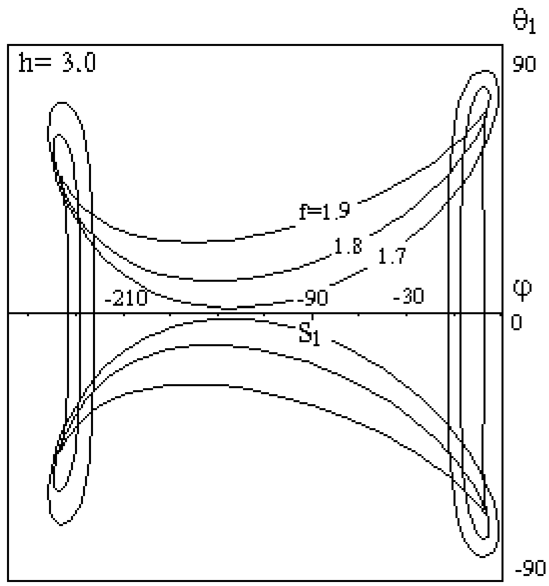

Figure 1.

When classifying the discovered trajectories, we assume the tracing of stationary points S_1 and S_2 on Poisson sphere, which are the analogues of libration points in celestial mechanics, as one of the criteria.

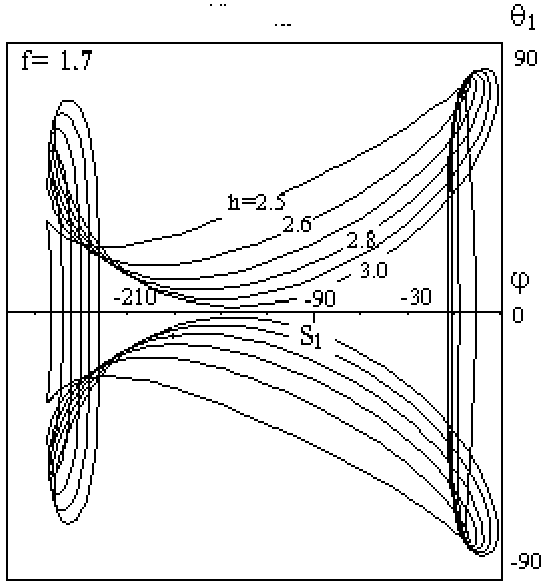

Figure 2.

Simply periodical trajectories were detected modified the constant energy integral $2.1 \leq h \leq 3.4$ and the constant area integral $1.4 \leq f \leq 2.0$. Their configurations are presented in fig.1 and fig 2. All this class trajectories are straight lines and are situated in the vicinity of the stationary point S_1.

In order the analysis of the results to be more suitable, the revealed periodical trajectories were recalculated for system (l) with corresponding going to Euler angles φ and θ. The value of the angle $\theta = \pi/2$ conforms the value $y_0 = 0$ of the reduced system, therefore in figures 1 and 2 the trajectories are presented in φ and $\theta_1 = \pi/2$ - θ coordinates. The values of angles are given in radians in the tables and in degrees - in figures.

The values of constant integrals of energy h and areas f, the initial value φ_0 and the value φ_1, for the second crossing of the trajectory in configuration space with abscissa axis, and the half-period $T/2$ quantity as well, corresponding to real time t are adduced in applied table with the aim of characterizing the class of the revealed simply periodical trajectories.

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Table 1.

nr.	h	f	φ_0	φ_1	T/2	ψ_1	a
1	2.10	1.40	1.32514	-2.84261	6.53097	10.87520	32.30044
2	2.20	1.50	1.38057	-3.05658	6.40267	10.75484	25.37029
3	2.30	1.50	1.26163	-2.91374	5.80668	10.32496	14.88485
4	2.30	1.60	1.46613	-3.39284	6.10181	10.35823	11.55669
5	2.40	1.50	1.16014	-2.84541	5.43212	10.07337	10.35813
6	2.40	1.60	1.31570	-3.11137	5.67844	10.23241	12.07613
7	2.50	1.60	1.20690	-3.00925	5.32770	10.01555	9.12742
8	2.50	1.70	1.38323	-3.39377	5.45798	9.94497	7.45313
9	2.60	1.60	1.10792	-2.95800	5.06730	9.85965	7.20013
10	2.60	1.70	1.25893	-3.20480	5.20555	9.91303	7.44512
11	2.70	1.60	1.01148	-2.93512	4.86082	9.74289	5.88298
12	2.70	1.70	1.15379	-3.12312	4.96945	9.80001	6.30109
13	2.70	1.80	1.31148	-3.49142	4.99664	9.61482	4.70137
14	2.80	1.70	1.05413	-3.08359	4.77588	9.70345	5.30023
15	2.80	1.80	1.20019	-3.32724	4.85047	9.68143	5.04258
16	2.90	1.70	0.95435	-3.07050	4.61348	9.62446	4.48731
17	2.90	1.80	1.09544	-3.25672	4.68231	9.63334	4.54377
18	2.90	1.90	1.21690	-3.66344	4.62595	9.31565	2.85933
19	3.00	1.70	0.85026	-3.07881	4.47384	9.55921	3.79275
20	3.00	1.80	0.99184	-3.22693	4.53305	9.57730	3.95930
21	3.00	1.90	1.12622	-3.48591	4.56120	9.48767	3.43478
22	3.10	1.80	0.88398	-3.22548	4.40203	9.52567	3.38409
23	3.10	1.90	1.01976	-3.42023	4.44079	9.49152	3.22613
24	3.20	1.80	0.76502	-3.25099	4.28593	9.48012	2.81330
25	3.20	1.90	0.90603	-3.40234	4.32409	9.46773	2.83248
26	3.20	2.00	0.99003	-3.72755	4.30942	9.29932	2.00737
27	3.30	1.80	0.61878	-3.31535	4.18195	9.44012	2.16267
28	3.30	1.90	0.77568	-3.42448	4.21700	9.43831	2.31915
29	3.30	2.00	0.87981	-3.66087	4.22943	9.35798	1.91986
30	3.40	1.90	0.58528	-3.52235	4.11943	9.40894	1.52456
31	3.40	2.00	0.69030	-3.71662	4.13923	9.36494	1.28415