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Corrigendum to "The multiplicative Zagreb co-indices on two graph operators" Bul. Acad. Ştiinţe Repub. Mold. Mat., 2016, no. 2(81), 18-26.

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Abstract. There is an error in the statement of Theorem 1 in the paper [1]. We give the correct statement and proof of the theorem.

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We follow the notations and symbols of [1]. In [1] in Theorem 1 the authors state that

$\overline{\prod}_1(T_{n,k}) = (2^{n^2+k^2+2nk-7n-7k+16})(5^{n+k-5})(3^{n+k-4})$. Here we prove that
$\overline{\prod}_1(T_{n,k}) = (2^{n^2+k^2+2nk-7n-7k+16})(5^{n+k-5})(3^{n+k-3})$ for $k \geq 2$ and
$\overline{\prod}_1(T_{n,1}) = (2^{n^2-5n+6})(5^{n-3})(3^{n-1})$.

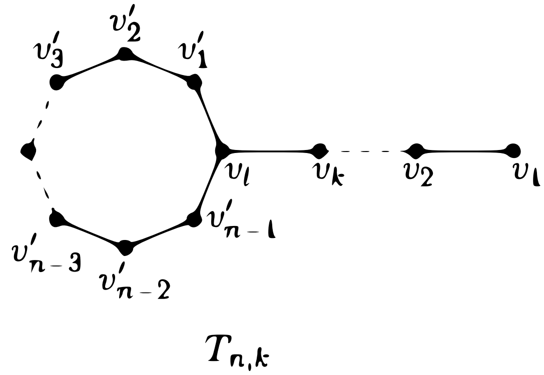
Theorem 1. *For the tadpole graph, the first multiplicative Zagreb co-indices satisfy the following equations:*
$\overline{\prod}_1(T_{n,k}) = (2^{n^2+k^2+2nk-7n-7k+16})(5^{n+k-5})(3^{n+k-3})$ *for* $k \geq 2$ *and*
$\overline{\prod}_1(T_{n,1}) = (2^{n^2-5n+6})(5^{n-3})(3^{n-1})$.

Proof. **Case 1:** $k \geq 2$

The tadpole graph $T_{n,k}$ contains $n+k-2$ vertices of degree 2, one vertex of degree 3 and a pendent vertex. The subdivision graph $S(T_{n,k})$ contains $n+k$ additional vertices of degree 2. In $T_{n,k}$, let v_l be a vertex of degree 3 and v'_1 and v'_{n-1} be the neighbors of v_l in the cycle C_n and v_k be the neighbor of v_l in the path P_{k+1}. Let v_1 be the pendent vertex in $T_{n,k}$. We calculate $\overline{\prod}_1[d_G(u) + d_G(v)]$:

1. Among the vertices in C_n.
2. From cycle C_n to the path P_{k+1}.
3. Among the vertices in the path P_{k+1}.

Sub-case I. In C_n, v'_1 and v'_{n-1} are non-adjacent with $n-3$ vertices of degree 2. Remaining $n-3$ vertices in C_n are non-adjacent with $n-4$ vertices of degree 2 and one vertex of degree 3. Also v_l is non-adjacent with $n-3$ vertices of degree 2. Hence in C_n, $\prod_1[d_G(u) + d_G(v)] = (4^{n^2-5n+6})(5^{2n-6})$. Since one edge is shared between a pair of vertices, $\overline{\prod}_1[d_G(u) + d_G(v)]$ in C_n is



$$\overline{\prod}_1[d_G(u)+d_G(v)] = \{(4^{n^2-5n+6})(5^{2n-6})\}^{(1/2)} = (2^{n^2-5n+6})(5^{n-3}). \qquad (1)$$

Sub-case II. From cycle C_n (where C_n is the cycle $(v_l, v'_1, v'_2, ..., v'_{n-1})$) to path P_{k+1} (where P_{k+1} is the path $(v_1, v_2, ..., v_k, v_l)$), all the $n-1$ vertices other than v_l in C_n are non-adjacent with $v_1, v_2, v_3, ..., v_k$. Also all of $n-1$ vertices except v_l in C_n are non-adjacent with $k-1$ vertices of degree 2 and one vertex of degree 1. Hence

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{(k-1)(n-1)})(3^{n-1}). \qquad (2)$$

Sub-case III. In the path P_{k+1}, the vertex v_l is non-adjacent with $k-2$ vertices of degree 2 and one vertex of degree 1. The vertex v_k is non-adjacent with $k-3$ vertices of degree 2 and one vertex of degree 1. The vertex v_j is non-adjacent with $k-4$ vertices of degree 2 and one vertex of degree 1 and one vertex of degree 3 for $3 \leq j \leq k-1$. Also the vertex v_2 has $k-3$ non-adjacent vertices of degree 2 and one vertex of degree 3. The vertex v_1 has $k-2$ non-adjacent vertices of degree 2 and one vertex of degree 3. Thus

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (4^{k^2-5k+8})(5^{2k-4})(3^{2k-4}).$$

Since one edge is shared between a pair of vertices,

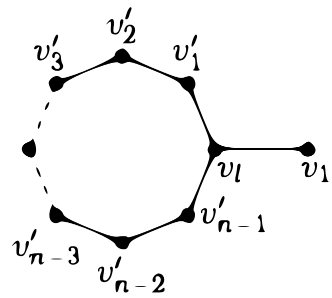
$$\overline{\prod}_1[d_G(u)+d_G(v)] = (2^{k^2-5k+8})(5^{k-2})(3^{k-2}). \qquad (3)$$

The product of equations (1), (2) and (3) implies that

$$\overline{\prod}_1(T_{n,k}) = (2^{n^2+k^2+2nk-7n-7k+16})(5^{n+k-5})(3^{n+k-3}).$$

Case 2: $k = 1$

The tadpole graph $(T_{n,1})$ contains $n-1$ vertices of degree 2, one vertex of degree 3 and a pendent vertex. Now we calculate $\overline{\prod}_1[d_G(u)+d_G(v)]$. v_1 is adjacent with v_l and v_1 is non-adjacent with remaining $n-1$ vertices of degree 2. v_l is adjacent with



$T_{n,1}$

v'_1 and v'_{n-1} (ignore v_1 as it is taken previous) and v_l is non-adjacent with remaining $n-3$ vertices of degree 2. v'_1 is adjacent with v'_2 (ignore v_l as it is taken previous) and non-adjacent with remaining $n-3$ vertices of degree 2 (ignore v_1 of degree 1 as it is taken previous). v'_2 is adjacent with v'_3 (ignore v'_1 as it is taken previous) and non-adjacent with remaining $n-4$ vertices of degree 2 (ignore v_1 of degree 1 and v_l of degree 3 as it is taken previous). For v'_3, v'_4... v'_{n-3} it is similar.

$$\overline{\prod}_1[d_G(u)+d_G(v)] = (5^{n-3})(3^{n-1})(4^{n-3}4^{n-4}4^{n-5}...4^1) = (2^{n^2-5n+6})(5^{n-3})(3^{n-1}).$$

$\square$

References

[1] Deldar M., Alaeiyan M. *The multiplicative Zagreb co-indices on two graph operators*. Bul. Acad. Ştiinţe Repub. Mold. Mat., 2016, no. 2(81), 18–26.

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