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Near-totally conjugate orthogonal quasigroups

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Abstract. The near-totally conjugate orthogonal quasigroups (near-*totCO*-quasigroups), i. e., quasigroups for which there exist five (but there are no six) pairwise orthogonal conjugates, are studied. We consider six types of such quasigroups, connection between them and prove that for any integer $n \geq 7$ which is relatively prime to 2, 3 and 5 there exist near-*totCO*-quasigroups of order n of any type. Three types of conjugate orthogonality graphs, associated with these quasigroups are characterized.

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1 Introduction

A quasigroup is an ordered pair (Q, A), where Q is a set and A is a binary operation, defined on Q, such that each of the equations $A(a, y) = b$ and $A(x, a) = b$ is uniquely solvable for any pair of elements a, b in Q. It is known that the multiplication table of a finite quasigroup defines a Latin square and six (not necessarily distinct) conjugates (or parastrophes) are associated with each quasigroup (Q, A) (Latin square): $A = {}^1A$, ${}^rA$, ${}^lA$, ${}^{rl}A$, ${}^{lr}A$, ${}^sA$, which are quasigroups, where ${}^{rl}A ={}^r({}^lA)$ and

$$^rA(x, y) = z \Leftrightarrow A(x, z) = y, \quad {}^lA(x, y) = z \Leftrightarrow A(z, y) = x, \quad {}^sA(x, y) = A(y, x).$$

Two quasigroups (Q, A) and (Q, B) are orthogonal ($A \perp B$) if the system of equations $\{A(x, y) = a,\ B(x, y) = b\}$ is uniquely solvable for all $a, b \in Q$.

A set $\Sigma = \{A_1, A_2, ..., A_n\}$ of quasigroups, defined on the same set, is orthogonal if any two quasigroups of this set are orthogonal.

The notion of orthogonality plays an important role in the theory of Latin squares, also in quasigroup theory and in distinct applications, in particular, in coding theory and cryptography. In addition, quasigroups that are orthogonal to some of their conjugates or two conjugates of which are orthogonal (known as conjugate orthogonal or parastrophic-orthogonal quasigroups) have a significant interest.

Many articles were devoted to the investigation of various aspects of conjugate orthogonal quasigroups. Recall some of them.

In [4–7, 9, 13], the spectrum of conjugate orthogonal quasigroups (Latin squares) was studied. Different identities associated with such orthogonality and related combinatorial designs were considered in [1, 4, 10].

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F. E. Bennett and Hantao Zhang [8] considered a problem related to the spectrum of Latin squares if each conjugate is required to be orthogonal to its transpose (to precisely its transpose among the other five conjugates) Latin square. In the paper [5], in particular, it is shown that for all $n > 5594$, with the possible exception of $n = 6810$, a Latin square with all distinct and pairwise orthogonal conjugates exists. The proof rests on several constructions of pairwise balanced designs of index one. In [6], F. E. Bennett improved this result, proving that such idempotent Latin squares exist for any order $n > 5074$.

In [7, 8, 12], connection between conjugate orthogonal Latin squares and graphs is considered. An orthogonal Latin square graph is one in which the vertices are Latin squares of the same order and on the same symbols, and two vertices are adjacent if and only if the Latin squares are orthogonal. In the article [7], *a graph of conjugate orthogonality* of a Latin square (a finite quasigroup) was considered, i. e., the graph the vertices of which are six conjugates of a Latin square and two vertices are connected if and only if the corresponding pair of conjugates is orthogonal.

The article [2] is devoted to the study of conjugate sets of a quasigroup and of quasigroups all conjugates of which are distinct (DC-quasigroups). In [3], the quasigroups all six conjugates of which form an orthogonal set were investigated.

In this paper we study quasigroups for which there exist five (but there are no six) pairwise orthogonal conjugates. We call such quasigroups near-totally conjugate orthogonal quasigroups (shortly, near-*totCO*-quasigroups), give some information about the spectrum of these quasigroups and characterize graphs, associated with them.

2 Totally and near-totally conjugate-orthogonal quasigroups

It is known that the number of distinct conjugates of a quasigroup can be 1, 2, 3 or 6 (see, for example, [11]).

A quasigroup (Q, A) is called *a totally conjugate-orthogonal quasigroup or a totCO-quasigroup* if all six its conjugates are pairwise orthogonal [3]. In this case the system of six conjugates of a quasigroup is an orthogonal set. Any conjugate of a *totCO*-quasigroup is also a *totCO*-quasigroup.

A quasigroup (Q, A) is called *a T-quasigroup* if there exist an abelian group $(Q, +)$, its automorphisms φ, ψ and an element $a \in Q$ such that $A(x, y) = \varphi x + \psi y + a$.

Let $\sigma \perp \tau$ mean that ${}^{\sigma}\!A \perp {}^{\tau}\!A$. It is evident that if $\sigma \perp \tau$, then $s\sigma \perp s\tau$.

The following theorem of [3] gives conditions for the orthogonality of pairs of conjugates for a T-quasigroup.

Theorem 1 [3]. *Let (Q, A) be a finite or infinite T-quasigroup of the form $A(x, y) = \varphi x + \psi y$. Then two its conjugates are orthogonal if and only if the following mappings corresponding to these conjugates:*

$(1 \perp l$ *or* $s \perp lr) \to \varphi + \varepsilon$, $\quad (r \perp rl) \to \varphi + \varepsilon$ *and* $\varphi - \varepsilon$,

$(1 \perp r$ *or* $s \perp rl) \to \psi + \varepsilon$, $\quad (l \perp lr) \to \psi + \varepsilon$ *and* $\psi - \varepsilon$,

$(1 \perp lr \ \textit{or} \ s \perp l) \to \varphi + \psi^2$, $\quad (1 \perp rl \ \textit{or} \ s \perp r) \to \varphi^2 + \psi$,
$(r \perp lr \ \textit{or} \ rl \perp l) \to \varphi - \psi$, $\quad (1 \perp s) \to \varphi - \psi$ *and* $\varphi + \psi$,
$(l \perp r \ \textit{or} \ lr \perp rl) \to \psi\varphi - \varepsilon$ *are permutations.*

In this theorem $(\varphi + \psi) : (\varphi + \psi)x = \varphi x + \psi x$ is an endomorphism of the abelian group of a T-quasigroup, ε is the identity permutation on Q.

Note that conditions of the theorem are valid also for T-quasigroups of the form $A(x, y) = \varphi x + \psi y + a$.

In [1], the following criterion for a $totCO$-T-quasigroup was established.

Theorem 2 [3]. *A T-quasigroup (Q, A): $A(x, y) = \varphi x + \psi y + a$ is a $totCO$-quasigroup if and only if all the following mappings*

$$\varphi + \varepsilon,\ \varphi - \varepsilon,\ \psi + \varepsilon,\ \psi - \varepsilon,\ \varphi^2 + \psi,\ \psi^2 + \varphi,\ \varphi - \psi,\ \varphi + \psi,\ \psi\varphi - \varepsilon$$

are permutations.

Corollary 1 [3]. *A T-quasigroup (Q, A): $A(x, y) = ax + by \pmod n$ is a $totCO$-quasigroup if and only if all elements*

$$a + 1,\ a - 1,\ b + 1,\ b - 1,\ a^2 + b,\ b^2 + a,\ a - b,\ a + b,\ ab - 1$$

modulo n are relatively prime to n.

In [1], it was proved that there exist infinite $totCO$-quasigroups. For finite quasigroups it is valid the following

Theorem 3 [3]. *For any $n = p_1^{k_1} p_2^{k_2} ... p_s^{k_s}$, where p_i, $i = 1, 2, ..., s$, are prime numbers not equal to 2, 3, 5 and 7, $k_i \geq 1$, there exists a $totCO$-quasigroup of order n.*

Consider quasigroups that are not $totCO$-quasigroups and five conjugates of which form an orthogonal set.

Definition 1. *A quasigroup (Q, A) is called near-totally conjugate orthogonal (a near-$totCO$-quasigroup) if it is not a $totCO$-quasigroup and there exist five its pairwise orthogonal conjugates.*

In [2], a quasigroup is called a distinct conjugate quasigroup or, shortly, a DC-quasigroup, if all its conjugates are distinct. It is evident that $totCO$-quasigroups and near-$totCO$-quasigroups are DC-quasigroups.

Let (Q, A) be a DC-quasigroup, then there are exactly six different subsets with five conjugates:

$$\Sigma_1 = \{l, r, rl, lr, s\},\ \ \Sigma_s = \{1, l, r, lr, rl\},\ \ \Sigma_l = \{1, r, rl, lr, s\},$$

$$\Sigma_{lr} = \{1, l, r, rl, s\},\ \ \Sigma_r = \{1, l, rl, lr, s\},\ \ \Sigma_{rl} = \{1, l, r, lr, s\}.$$

The index σ of Σ_σ indicates the missing conjugate in a set.

Say that a near-$totCO$-quasigroup (Q, A) has *the conjugate orthogonality type or the CO-type* Σ_1 ($\Sigma_s, \Sigma_l, \Sigma_{lr}, \Sigma_r$ or Σ_{rl}) if the respective set of conjugates is orthogonal.

The following statement shows that there exist connections between CO-types of a quasigroup.

Proposition 1. *A near-totCO-quasigroup has the CO-type Σ_1 if and only if it has the CO-type Σ_s.*

A near-totCO-quasigroup has the CO-type Σ_l if and only if it has the CO-type Σ_{lr}.

A near-totCO-quasigroup has the CO-type Σ_r if and only if it has the CO-type Σ_{rl}.

Proof. Let the set Σ_1 be orthogonal, then it is easy to see that the set ${}^s\Sigma_1 = \{sl,\, sr,\, srl,\, slr,\, ss\} = \{lr,\, rl,\, l,\, r,\, 1\} = \Sigma_s$ is also orthogonal since the commutation of orthogonal operations retains their orthogonality. Analogously we obtain that ${}^s\Sigma_l = \Sigma_{lr}$ and ${}^s\Sigma_r = \Sigma_{rl}$. Note that $s = rlr = lrl$ and $\sigma\sigma = 1$ for any conjugate.

The converse statement is obvious. □

Corollary 2. *If a quasigroup is a near-totCO-quasigroup, then there exists an orthogonal set of five conjugates, containing this quasigroup.*

Indeed, in all cases at least one of two conjugate sets of Proposition 1 contains the initial quasigroup.

Proposition 2. *Any conjugate of a near-totCO-quasigroup is also a near-totCO-quasigroup.*

Proof. Let a near-*totCO*-quasigroup (Q, A) have the CO-type $\Sigma_1(A) = \{l,\, r,\, rl,\, lr,\, s\}$. Then any conjugate of this set is a near-*totCO*-quasigroup since it is contained in this orthogonal set, and the rest four conjugates can be considered as conjugates of this conjugate. For example, for the conjugate ${}^lA$ we have

$$\Sigma_1({}^lA) = \{l,\, r = (rl)l,\, (r)l,\, lr = (s)l,\, s = (lr)l\}.$$

Thus ${}^lA$ is included in an orthogonal set of its five conjugates.

Let a near-*totCO*-quasigroup (Q, A) have the CO-type $\Sigma_l = \{1,\, r,\, rl,\, lr,\, s\}$. As above, prove that the conjugates r, rl, lr, s are near-*totCO*-quasigroups. For the conjugate ${}^lA$ use the CO-type Σ_{lr} which by Proposition 1 is also a CO-type of (Q, A).

Analogously consider the conjugates of the sets Σ_r and Σ_{rl}. □

Theorem 4. *If a quasigroup $(Q,\, A)$ is not a totCO-quasigroup, then*

– the set Σ_1 (Σ_s) of its conjugates is orthogonal if and only if all pairs of conjugates, except the pair $(1, s)$, are orthogonal;

– the set Σ_l (Σ_{lr}) is orthogonal if and only if all pairs of conjugates, except the pair $(l,\, lr)$, are orthogonal;

– the set Σ_r (Σ_{rl}) is orthogonal if and only if all pairs of conjugates, except the pair $(r,\, rl)$, are orthogonal.

Proof. Using Proposition 1 and taking into account the orthogonal pairs of conjugates of Σ_1 and Σ_s (of Σ_l and Σ_{lr}; of Σ_r and Σ_{rl}), we will establish that there are exactly 14 orthogonal pairs of 15 possible pairs of conjugates and only the pair $(1,\, s)$ (the pair $(l,\, lr)$, the pair $(r,\, rl)$ respectively) is missing.

It is easy to check the converse statement. $\square$

Note that from this theorem it follows that a near-$totCO$-quasigroup can not have CO-types simultaneously from two different pairs of connected CO-types. Otherwise it is a $totCO$-quasigroup.

Corollary 3. *A $T-$quasigroup $(Q,\, A)$: $A(x,\, y) = \varphi x + \psi y + a$ that is not a $totCO$-quasigroup is a near-$totCO$-quasigroup*

– of the type Σ_1 (Σ_s) if and only if all mappings of Theorem 2, except the unique mapping $\varphi + \psi$, are permutations;

– of the type Σ_l (Σ_{lr}) if and only if all mappings of Theorem 2, except the unique mapping $\psi - \varepsilon$, are permutations;

– of the type Σ_r (Σ_{rl}) if and only if all mappings of Theorem 2, except the unique mapping $\varphi - \varepsilon$, are permutations.

Proof. Show that the corresponding conditions of Theorem 4 and the corollary for T-quasigroups are equivalent. By Theorem 1, two permutations $\varphi+\varepsilon$, $\varphi-\varepsilon$ correspond to orthogonality of the pair $(1, s)$. But the mapping $\varphi - \varepsilon$ is a permutation since $r \perp lr$. Hence, only the mapping $\varphi + \varepsilon$ is not a permutation. It is easy to see that the converse statement is valid by Theorem 1.

Analogously, using Theorem 1, we obtain the pointed out conditions for the pairs $(l,\, lr)$ and $(r,\, rl)$. $\square$

Corollary 4. *An abelian group is not a near-$totCO$-quasigroup (a $totCO$-quasigroup).*

Indeed, an abelian group $(Q, +)$ is a T-quasigroup $A(x, y) = \varphi x + \psi y + a$ with $\varphi = \psi = \varepsilon$, $a = 0$, so for it $\varphi - \varepsilon = \psi - \varepsilon = \varphi - \psi = \varphi\psi - \varepsilon = 0$, where 0 is the zero of the endomorphism ring of the group $(Q, +)$. By Corollary 3 (by Theorem 2) the T-quasigroup is not a near-$totCO$-quasigroup (a $totCO$-quasigroup). $\square$

Corollary 5. *Among T-quasigroups there are not idempotent near-$totCO$-quasigroups of the CO-type Σ_1 (Σ_s).*

Indeed, if a T-quasigroup $A(x, y) = \varphi x + \psi y + a$ is idempotent, then $A(x, x) = x = \varphi x + \psi x + a = R_a^+(\varphi + \psi)x$, where $R_a^+ x = x + a$, whence the mapping $\varphi + \psi$ is a permutation. $\square$

Proposition 3. *A $T-$quasigroup $(Q,\, A)$: $A(x,\, y) = ax + by \pmod n$ that is not a $totCO-$quasigroup, is a near-$totCO$-quasigroup*

– of the type Σ_1 (Σ_s) if and only if all mappings of Theorem 2, except the unique mapping $x \to (a + b)x \pmod n$, are permutations;

– of the type Σ_l (Σ_{lr}) if and only if all mappings of Theorem 2, except the unique mapping $x \to (b - 1)x \pmod n$ are permutations;

– of the type Σ_r (Σ_{rl}) if and only if all mappings of Theorem 2, except the unique mapping $x \to (a - 1)x \pmod n$ are permutations.

Indeed, for a quasigroup of such form $(\varphi-\varepsilon)x = (L_a-\varepsilon)x = (a-1)x$; $(\psi-\varepsilon)x = (L_b-\varepsilon)x = (b-1)x$ and $(\varphi+\psi)x = (L_a+L_b)x = (a+b)x$, where $L_a x = ax$. $\square$

Corollary 6. *A $T-$ quasigroup $(Q, A) : A(x, y) = ax + by \pmod n$ is a near-totCO-quasigroup if and only if all numbers of Corollary 1 are relatively prime to n, except the single number of $a-1$, $b-1$ or $a+b$.*

Theorem 5. *For any integer $n \geq 7$ that is prime to 2, 3 and 5 there exists a near-totCO-quasigroup of every of six CO-types.*

Proof. Let $\overline{a}$ be an element a modulo n, (m, n) be the greatest common divisor of m and n. Consider the quasigroup (Q, A): $A(x, y) = x + 3y \pmod n$ (here $(3, n) = 1$). Check the conditions of Proposition 3 for this quasigroup: $(a+1)x = 2x$, $(a-1)x = 0x$, $(b+1)x = 4x$, $(b-1)x = 2x$, $(a^2+b)x = 4x$, $(b^2+a)x = 10x$, $(a-b)x = -2x$, $(a+b)x = 4x$, $(ab-1)x = 2x$ modulo n. Since $n \geq 7$ the mappings $2x$, $4x$, $10x$, $-2x$ are permutations if n is relatively prime to 2, 3 and 5. Note that if $(2, n) = 1$, then $(-2, n) = (n-2, n) = 1$.

Let n be relatively prime to 2, 3 and 5, then $n \neq 10$ and $n < 10$ only for $n = 7$. In this case $\overline{10} = 3$ and $(3, 7) = 1$. If $n > 10$, then $\overline{10} = 10$ and $(10, n) = 1$ as n is relatively prime to 2 and 5. Hence, only the mapping $(a-1)x$ is not a permutation and by Proposition 3, the quasigroup $A(x, y) = x + 3y \pmod n$ is a near-*totCO*-quasigroup of the CO-type Σ_r (Σ_{rl}) for any n, relatively prime to 2, 3 and 5.

Analogously, the quasigroup (Q, B): $B(x, y) = 3x + y \pmod n$, where $(3, n) = 1$ is a quasigroup of the CO-type Σ_l (Σ_{lr}) as only the mapping $(b-1)x = 0x$ is not a permutation.

The quasigroup (Q, C): $C(x, y) = 3x - 3y \pmod n$, where $(3, n) = 1$, is a quasigroup of the CO-type Σ_1 (Σ_s) since $(a+1)x = 4x$, $(a-1)x = 2x$, $(b+1)x = -2x$, $(b-1)x = -4x$, $(a^2+b)x = 6x$, $(b^2+a)x = 12x$, $(a-b)x = 6x$, $(a+b)x = 0x$, $(ab-1)x = -10x$.

Let n be relatively prime to 2, 3 and 5, then $n \neq 12$ and $n < 12$ only for $n = 7, 11$. These numbers are prime, so $(\overline{12}, n) = 1$.

If $n > 12$, then $\overline{12} = 12$ and $(12, n) = 1$ as n is relatively prime to 2 and 3. Thus only the mapping $x \to (a+b)x = 0x$ is not a permutation. By Proposition 3, (Q, C) is a quasigroup of the CO-type Σ_1 (Σ_s). $\square$

Corollary 7. *For any $n = p_1^{k_1} p_2^{k_2} ... p_s^{k_s}$, where p_i is a prime number, $p_i \neq 2, 3, 5$, $k_i \geq 1$, $i = 1, 2, ..., s$, $s \geq 1$, and for any of six CO-types there exists a near-totCO-quasigroup of order n.*

The computer research showed that near-*totCO*-quasigroups of order $n \leq 5$ do not exist among T-quasigroups of the form $A(x, y) = ax + by \pmod n$.

Besides the quasigroup which was used for the proof of Theorem 5, there exist other T-quasigroups with analogous properties.

Proposition 4. *The quasigroups (Q, A), (Q, B), (Q, C) : $A(x, y) = x + cy \pmod n$, $B(x, y) = cx + y \pmod n$, $C(x, y) = cx - cy \pmod n$, $c > 1$, of odd* *order n are near-totCO-quasigroups if and only if the numbers $c-1$, $c+1$, c^2+1 modulo n are relatively prime to n.*

Proof. At first note that the element c is relatively prime to n since A, B, C are quasigroups, so $(2c, n) = 1$ and the mapping $2x$ is a permutation in a group of odd order. After that it is easy to check that under these conditions all mappings of Corollary 1, except the mapping $(a-1)x = (1-1)x$ for the quasigroup A (except the mapping $(b-1)x = (1-1)x$ for the quasigroup B and except the mapping $(a+b)x = (c-c)x$ for the quasigroup C), are permutations. So by Corollary 6, these quasigroups are near-*totCO*-quasigroups. Moreover, by Proposition 3, the quasigroups A, B, C have the CO-types Σ_r and Σ_{rl}; Σ_l and Σ_{lr}; Σ_1 and Σ_s respectively.

Conversely, if the quasigroup A (B, C) is a near-*totCO*-quasigroup, then by Proposition 3, for each quasigroup all numbers, pointed out in Proposition 4 are relatively prime to n. □

Example. Consider the quasigroup $(Q,\, A) :\ A(x,\, y) = x + 2y \pmod 7$. For this quasigroup the numbers $c-1 = 1$, $c+1 = 3$, $c^2+1 = 5$ modulo 7 are relatively prime to 7. By Proposition 4, it is a near-*totCO*-quasigroup of the CO-types Σ_r and Σ_{rl}. Its conjugates (modulo 7) are:

$$A(x,\, y) = x + 2y,\ {}^{r}A(x, y) = -4x + 4y,\ {}^{l}A(x, y) = x - 2y,$$

$${}^{lr}A(x, y) = -2x + y,\ {}^{rl}A(x, y) = 4x - 4y,\ A^{*}(x, y) = 2x + y.$$

It is easy to check directly that all pairs of different conjugates, except the pair $({}^{r}A, {}^{rl}A)$, are orthogonal.

In the article [7], *the graph of conjugate orthogonality* of a Latin square (a finite quasigroup) was considered, i. e., the graph the vertices of which are six conjugates of a Latin square and two vertices are connected if and only if the corresponding pair of conjugates is orthogonal. It is evident that the complete graph K_6 of conjugate orthogonality corresponds to a *totCO*-quasigroup.

We call the graph of conjugate orthogonality of a quasigroup *near-complete* if its complement (with respect to the complete graph K_6) contains a single edge. Such graph contains exactly 14 edges.

From Theorem 4 and Corollary 7 the following statements immediately follow for graphs of conjugate orthogonality.

Theorem 6. *A near-totCO-quasigroup of the CO-type Σ_1 (Σ_s) corresponds to the near-complete graph of conjugate orthogonality without the edge $(1, s)$.*

A near-totCO-quasigroup of the CO-type Σ_l (Σ_{lr}) corresponds to the near-complete graph of conjugate orthogonality without the edge (l, lr).

A near-totCO-quasigroup of the CO-type Σ_r (Σ_{rl}) corresponds to the near-complete graph of conjugate orthogonality without the edge (r, rl).

Proposition 5. *For every $n = p_1^{k_1} p_2^{k_2} ... p_s^{k_s}$, where p_i is a prime number, $p_i \neq 2, 3, 5$, $k_i \geq 1$, $i = 1, 2, ..., s$, $s \geq 1$, there exists a Latin square (a quasigroup) of order n, corresponding to a near-complete graph of conjugate orthogonality.*

Recall that two graphs are isomorphic if the vertices of every graph can be numbered such that the vertices in one graph are connected if and only if in the second graph the vertices with the same numbers are neighboring. Isomorphic graphs have the same structure.

It is easy to see that three near-complete graphs corresponding to different CO-types of near-$totCO$-quasigroups are isomorphic.

References

[1] Belousov V. D. *Parastrophic-orthogonal quasigroups.* Quasigroups and Related Systems, 2005, **13**, No. 1, 25–73.

[2] Belyavskaya G. B., Popovich T. V. *Conjugate sets of loops and quasigroups. DC-quasigroups.* Bul. Acad. Ştiinţe Repub. Moldova. Mat., 2012, No. 1(59), 21–32

[3] Belyavskaya G. B., Popovich T. V. *Totally conjugate orthogonal quasigroups and complete graphs.* Journal of Mathematical Sciences, 2012, **185**, No. 2, 184–191.

[4] Bennett F. E. *The spectra of a variety of quasigroups and related combinatorial designs.* Discrete mathematics, 1989, **77**, 29–50.

[5] Bennett F. E. *Latin squares with pairwise orthogonal conjugates.* Discrete mathematics, 1981, **36**, 117–137.

[6] Bennett F. E. *On conjugate orthogonal idempotent Latin squares.* Ars. combinatorica, 1985, **19**, 37–50.

[7] Bennett F. E., Mendelsohn N. S. *Conjugate orthogonal Latin square graphs.* Congressus Numerantium, 1979, **23**, 179–192.

[8] Bennett F. E., Zhang Hantao. *Latin squares with self-orthogonal conjugates.* Discrete mathematics, 2004, **284**, 45–55.

[9] Chaffer R. A., Lieberman D. J., Smith D. D. *The number of orthogonal conjugates of a quasigroup.* Congressus Numerantium, 1982, **35**, 169–180.

[10] Evans T. *Algebraic structures associated with Latin squares and orthogonal arrays.* Proc. Conf. Algebraic Aspects of Combinatorics, Congressus Numerantium, 1975, **13**, 31–52.

[11] Lindner C. C., Steedly D. *On the number of conjugates of a quasigroup.* Algebra Univ., 1975, **5**, 191–196.

[12] Lindner C. C., Mendelsohn E., Mendelsohn N. S., Wolk B. *Orthogonal Latin square graphs.* J. Graph Theory, 1979, **3**, 325–328.

[13] Phelps K. T. *Conjugate orthogonal quasigroups.* J. Combin. Theory (A), 1978, **25**, 117–127.

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