BULETINUL ACADEMIEI DE ŞTIINŢE
A REPUBLICII MOLDOVA. MATEMATICA
Number 3(58), 2008, Pages 89–98
ISSN 1024–7696

About Division of d-Convex Simple Graphs in M-Prime Graphs

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Abstract. In this article we research the structure of d-convex simple graphs in order to extend the already known classes of graphs of this type. We do this using some new operations and new graphs. We introduce the notion of M-prime graphs and split all d-convex simple graphs into M-prime graphs using the M operation. After that we describe all M-prime graphs we know.

Mathematics subject classification: 52A30, 05C75.
Keywords and phrases: d-convex simple graph, M operation, M-prime graph, special classes of graphs.

1 Preliminary Considerations

After we obtained an iterative method of the characterisation of all d-convex simple graphs [3], we observe that this method allow us to construct d-convex simple graphs, which are complicated enough, but it says few things about the structure of these graphs, what they look like and how diverse they are. Therefore, to learn this structure, which would allow us to resolve some application problems, like in [4], we start to study these graphs via special classes, to be precise we want to extend the already known classes of d-convex simple graphs from [5, 6]. So we introduce a new operation M [1, 2], that is algebraic on all known classes of these graphs and allows us to do this extensions by using only one new graph. In this article, using the M-operation and some new operations on this set of graphs, we will define new classes of d-convex simple graphs, which extend the already known classes of d-convex simple graphs and have visible structure, our goal being to characterise as many d-convex simple graphs as it is possible.

Definition 1. [5, 6] An undirected graph $G = (X, U)$ is called *d-convex simple* if any subset of vertexes $A \subset X$, $2 < |A| < |X|$ is not d-convex.

Let us denote by $\Gamma(x)$ the **neighbourhood** of vertex x, i. e. $\Gamma(x) = \{y \in X | x \sim y\}$. We will say that vertex x **dominates** the vertex y if $\Gamma(x) \supset \Gamma(y)$ and vertex y is called a **copy** for vertex x ($x \neq y$), in graph $G = (X; U)$ if $\Gamma(x) = \Gamma(y)$ [5].

Definition 2. [5] The subset of vertexes D is called *dominating* in graph $G = (X, U)$ if for $\forall x \in X$, $\exists y \in D$ such that y dominates x, where it is possible that $y = x$.

*The research of the author was supported by SCSTD of ASM grant 07411.08 INDF and MRDA/CRDF Grant CERIM 10006-06.

In [5] it is shown that all dominating sets that are minimal by inclusions, of a graph G are isomorphic and each of them generates the same subgraph G_0, called the **atom** of graph G. To construct G_0 we have first to find the sets:

$$S = \{x \in X : \forall y \in X \Rightarrow \Gamma(x) \nsubseteq \Gamma(y)\};$$

$$R = \{x \in X \backslash S : \forall y \in X \Rightarrow \Gamma(x) \not\subset \Gamma(y)\}.$$

Then for $\forall x \in R$ form the set $R(x) = \{x\} \cup \{y \in R : \Gamma(x) = \Gamma(y)\}$. By this way R is divided into classes of equivalence. G_0 is formed from S and one vertex from each class of equivalence. For each vertex $x_0 \in G_0$ we can find a vertex $x \in G$ such that x corresponds to x_0, because G_0 is a copy of a subgraph of G. We denote by $L(G,\, G_0)$ a new graph that is obtained from G, G_0 and the following edges: for each vertex $x_0 \in G_0$ we will add all edges between x_0 and all vertexes from $\Gamma(x)$. It is easy to see that in the graph $L(G,\, G_0)$ the pair x, x_0 will be adjacent to the same vertexes, so the pairs of such kind will be pairs of copies. The next theorem is true:

Theorem 1. *[5, 6] If G is a connected graph, without cycles of length three (called triangles), then the graph $L(G,\, G_0)$ is d-convex simple, where G_0 is the atom of the graph G.*

Let G_1 and G_2 be two d-convex simple graphs, which have one pair of copies $x_1,\, x_2 \in G_1$ and $y_1,\, y_2 \in G_2$. Let us denote by $M^{x_1=y_1}_{x_2=y_2}(G_1,\, G_2)$ the graph obtained from G_1 and G_2 by pasting together x_1 with y_1 and x_2 with y_2. The new graph $G = M^{x_1=y_1}_{x_2=y_2}(G_1,\, G_2)$ contains two vertexes less than the union of graphs G_1 and G_2 and as many edges as have G_1 and G_2 together (Fig. 1). We will write $G = M(G_1,\, G_2)$ if we know the pairs of vertexes that participate in forming the new graph G.

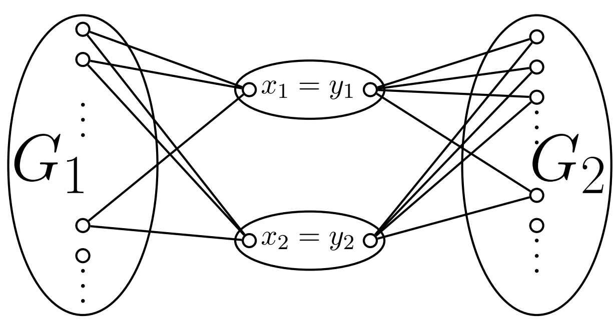


Figure 1.

Theorem 2. *[1, 2] If G_1 and G_2 are two d-convex simple graphs, where one pair of copies, $x_1,\, x_2 \in G_1$ and $y_1,\, y_2 \in G_2$ exists, then the graph $G = M^{x_1=y_1}_{x_2=y_2}(G_1,\, G_2)$ is also d-convex simple.*

From this theorem it results that the operation M introduced above is an algebraic operation on the set of d-convex simple graphs. In [1] this operation M is studied on some known classes of d-convex simple graphs [5] namely:

1. $\mathcal{A}$ is the set of all d-convex simple graphs without cycles of length 3, where each vertex is dominated by other;
2. $\mathcal{F}$ are graphs without cycles of length 3 and without generated subgraphs F (Fig. 2a);
3. $\mathcal{H}_1$ are hereditary-modular graphs, i.e. bipartite graphs where each isometric cycle is of length 4;
4. $\mathcal{H}_2$ are chordal graphs, i.e. bipartite graphs where each generated cycle is of length 4;
5. $\mathcal{H}_3$ are hereditary by distance graphs, i.e. graphs without cycles of length 3 and where each generated connected subgraph is isometric;
6. $\mathcal{P}$ is the set of d-convex simple and planar graphs.

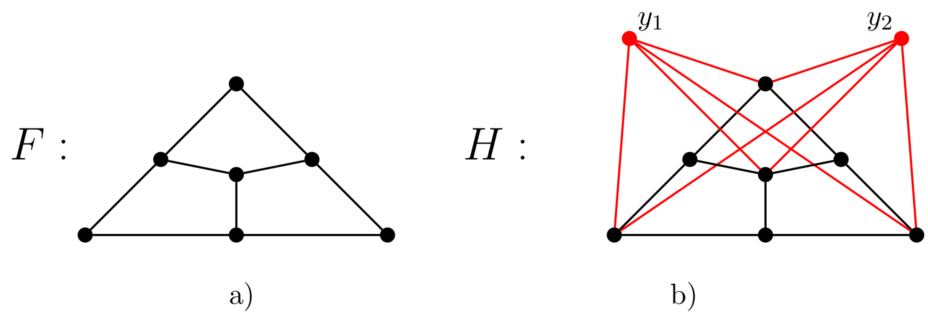


Figure 2.

Let $S\mathcal{F}$, $S\mathcal{H}_1$, $S\mathcal{H}_2$, $S\mathcal{H}_3$ be all d-convex simple graphs from classes of graphs $\mathcal{F}$, $\mathcal{H}_1$, $\mathcal{H}_2$, $\mathcal{H}_3$ respectively. The next lemmas are true:

Lemma 1. *[5] If G is a graph from the class $\mathcal{A}$, then G is d-convex simple.*

Lemma 2. *[5] If G is a d-convex simple graph without generated subgraphs F (Fig. 2a), then G is from $\mathcal{A}$.*

Also, in [5] is proved next relation:

$$\mathcal{P} \subset S\mathcal{H}_3 \subset S\mathcal{H}_2 \subset S\mathcal{H}_1 \subset S\mathcal{F} \subset \mathcal{A}.$$

We have to say that reverse affirmations of Lemmas 1, 2 are false, because a d-convex simple graph that is not in $\mathcal{A}$ is the graph H illustrated in Fig. 2b and a d-convex simple graph that contains F as a generated subgraph and belongs to the class $\mathcal{A}$ is the graph $L(F, F_0)$.

Theorem 3. *[5] Let G be a locally finite graph then:*

1. *$G \in \mathcal{A}$ if and only if $G = L(\Gamma, \Gamma_0)$, where Γ is a connected graph without cycles of length 3, Γ_0 is the atom of Γ;*

2. *$G \in S\mathcal{F}$ if and only if $G = L(\Gamma, \Gamma_0)$, where $\Gamma \in \mathcal{F}$;*
3. *$G \in S\mathcal{H}_i$ if and only if $G = L(\Gamma, \Gamma_0)$, where $\Gamma \in \mathcal{H}_i$, $i = 1, 2, 3$;*
4. *$G \in \mathcal{P}$ if and only if $G = L(\Gamma, \Gamma_0)$, where Γ is a tree with at least 3 vertexes.*

From this theorem and L operation it results that the graphs of classes $\mathcal{P}$, S$\mathcal{H}_1$, S$\mathcal{H}_2$, S$\mathcal{H}_2$, S$\mathcal{F}$, $\mathcal{A}$ have at least one pair of vertexes copies and then we can apply the M operation to them.

Theorem 4. *[1] For any two finite graphs G_1 and G_2 are true the next affirmations:*

1. *If $G_1, G_2 \in \mathcal{A}$, then $G = M(G_1, G_2) \in \mathcal{A}$;*
2. *If $G_1, G_2 \in S\mathcal{F}$, then $G = M(G_1, G_2) \in S\mathcal{F}$;*
3. *If $G_1, G_2 \in S\mathcal{H}_i$, then $G = M(G_1, G_2) \in S\mathcal{H}_i$, $i = 1, 2, 3$;*
4. *If $G_1, G_2 \in \mathcal{P}$, then $G = M(G_1, G_2) \in \mathcal{P}$;*
5. *If G_1, G_2 are two d-convex simple and bipartite graphs which have at least one pair of copies, then $G = M(G_1, G_2)$ is also d-convex simple and bipartite.*

In other words, theorem 6 asserts that the introduced operation M is algebraic on all mentioned classes, where the class $\mathcal{A}$ is the vastest. Lemma 4 asserts that all d-convex simple graphs without generated subgraph F are from $\mathcal{A}$. It results that a new class of d-convex simple graphs should have only graphs with F as a subgraph and some vertexes that are not dominated.

2 Extensions of Classes of d-Convex Simple Graphs

Let us denote by $\mathcal{C}$ a class of d-convex simple graphs, for example one of classes we have mentioned above .

Let G be a d-convex simple graph, not from $\mathcal{C}$, which has at least one pair of copies vertexes, because we need that G could participate in the operation M with other graphs. For example if we consider that $\mathcal{C} = \mathcal{A}$ then as G the graph H (Fig. 2b) can be toked.

Definition 3. The set of all graphs that could be obtained from the graph G and graphs of set $\mathcal{C}$, by using the M operation a finite number of times, is called *the extension of class $\mathcal{C}$ by graph G* and denoted $\mathcal{C}$[G].

The next properties are true:

1. $\mathcal{C}[G]$ is a class of d-convex simple graphs;
2. $\mathcal{C} \subset \mathcal{C}[G]$;
3. If $\mathcal{C}_1 \subset \mathcal{C}_2$, then $\mathcal{C}_1[G] \subset \mathcal{C}_2[G]$;

4. If $\mathcal{C}_1 \subset \mathcal{C}_2$ and $G \notin \mathcal{C}_2$, then $\mathcal{C}_1[G] \not\subset \mathcal{C}_2$.

Now we can form the extensions of known classes of d-convex simple graphs by graph H (Fig. 2b). We obtain $\mathcal{P}[H]$, $S\mathcal{H}_i[H]$, $i = 1, 2, 3$, $S\mathcal{F}[H]$, $\mathcal{A}[H]$. The next relation is true:

$$\mathcal{P}[H] \subset S\mathcal{H}_3[H] \subset S\mathcal{H}_2[H] \subset S\mathcal{H}_1[H] \subset S\mathcal{F}[H] \subset \mathcal{A}[H].$$

Moreover, it is also true that $\mathcal{A} \subset \mathcal{A}[H]$, so the class $\mathcal{A}[H]$ is larger than all classes of d-convex simple graphs known by now.

But the graph H is not the unique graph that can make extensions, and any other graph, that has the same properties generate with $\mathcal{A}$ new extensions. We can also make extension of extension of some classes of graphs. Let σ be a set of d-convex simple graphs and $\mathcal{C}$ be a class of d-convex simple graphs such that the graphs of the set σ are not from class $\mathcal{C}$, then:

Definition 4. The set of all graphs that could be obtained from the graphs of the set σ and graphs of set $\mathcal{C}$, by using the M operation a finite number of times, is called *the extension of class* $\mathcal{C}$ *by the set* σ and denoted $\mathcal{C}[\sigma]$.

The next properties are true:

1. $\mathcal{C} \subset \mathcal{C}[\sigma] \subset \mathcal{G}$, where $\mathcal{G}$ is the set of all d-convex simple graphs;

2. $\mathcal{C}[\sigma_1 \cup \sigma_2] = \mathcal{C}[\sigma_1][\sigma_2] = \mathcal{C}[\sigma_2][\sigma_1]$.

Definition 5. We will say that the d-convex simple graph G is *divisible* with respect to the M operation if there exist two d-convex simple graphs G_1 and G_2 such that $G = M(G_1, G_2)$. In this case the graphs G_1 and G_2 will be called *divisors* of the graph G.

Definition 6. The d-convex simple graph G is called *M-prime* if it is not divisible with respect to the M operation.

It is easy to see that the graph H (Fig. 2b) is a M-prime graph.

Theorem 5. *The d-convex simple graph G is divisible with respect to the M operation if and only if there exists a pair of copies vertexes z_1 and z_2 in G such that as result of the elimination of the vertexes z_1, z_2 from G, we obtain an unconnected graph.*

Proof. Necessity: Let G be a d-convex simple graph that is divisible with respect to the M operation, then by the definition of divisibility there are d-convex simple graphs G_1, G_2, with pairs of copies vertexes x_1, x_2 and y_1, y_2 respectively, such that $G = M^{x_1=y_1}_{x_2=y_2}(G_1, G_2)$. As result of the elimination from the graph G of the vertexes $z_1 = x_1 = y_1$, $z_2 = x_2 = y_2$, the obtained graph is obviously unconnected.

Sufficiency: Let G be a d-convex simple graph, where there exists a pair of vertexes copies z_1, z_2, such that as result of the elimination of the vertexes z_1, z_2

from G, we obtain a graph with two components, not necessarily connected. Let us denote by G_1 and G_2 each of components and make the next changes: in the first component we add two vertexes x_1, x_2, and all vertexes from G_1 which were adjacent with z_1, z_2 in G now will be adjacent with x_1, x_2; in the second component we also add two vertexes y_1, y_2 and make the same thing, i.e. all vertexes from G_2 which were adjacent with z_1, z_2 in G now will be adjacent with y_1, y_2. Of course $G = M_{x_2=y_2}^{x_1=y_1}(G_1, G_2)$. It remains to prove that the graphs G_1 and G_2 are d-convex simple. Let us show that G_1 is d-convex simple, the fact that G_2 is d-convex simple can be proved by analogy. First we want to show that $d - conv_{G_1}(\{x_1, x_2\}) = X_{G_1}$. Indeed, if we construct d-convex hull in G of any two vertexes v_1 and v_2 that are not from G_1, then there must be obligatory the vertexes z_1 and z_2, which would attract in this hull all vertexes from G_1. From this result

$$d - conv_{G_1}(\{x_1, x_2\}) = X_{G_1}. \qquad (*)$$

Let now x, y be two vertexes from G_1 at distance two. As above, because G is d-convex simple we have

$$d - conv_G(\{x, y\}) = \cup_{i=0}^{\infty} B_i = X_G.$$

Results that $\exists k \geq 0$ such that $z_1, z_2 \in B_k$ and $z_1, z_2 \notin B_{k-1}$ and all vertexes from B_k, except z_1, z_2, are from G_1, because after the elimination of z_1, z_2 from G an unconnected graph remained. We construct by the same way the convex hull of vertexes x, y in G_1 until we come to the set B_k, where instead of vertexes z_1, z_2, we have x_1, x_2. So we have that $\{x_1, x_2\} \subset d - conv(\{x, y\}$, from $(*)$ it results that

$$G_1 = d - conv_{G_1}(\{x_1, x_2\}) \subseteq d - conv_{G_1}(\{x, y\}).$$

The reverse inclusions is obvious so we have that G_1 is a d-convex simple graph.

□

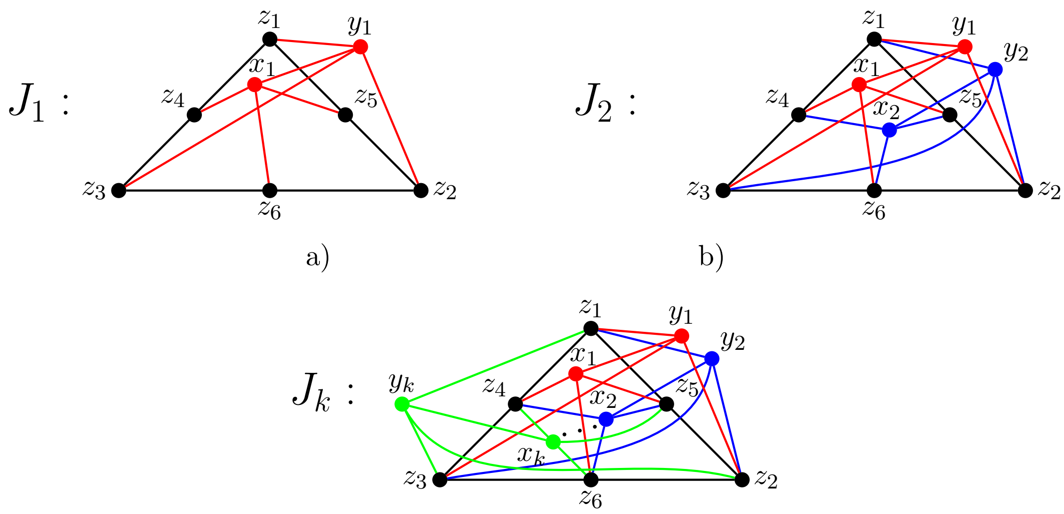
From this theorem follows the next corollary:

Corollary. *Decomposition of an arbitrary d-convex simple graph into M-prime graphs is unique.*

3 The Sets of M-Prime Graphs

Let us denote by $\mathcal{B}$ the set of all M-prime graphs from $\mathcal{G} \backslash \mathcal{A}$, by $\mathcal{B}_1$ those graphs from $\mathcal{B}$ which have at least one pair of vertexes copies, and denote by $\mathcal{B}_2$ the rest graphs from $\mathcal{B}$, i.e. those graphs where don't exist pairs of copies vertexes. We have $\mathcal{B}_1 \neq \emptyset$ because $H \in \mathcal{B}_1$ (Fig. 2b). Let us see that $\mathcal{B}_2 \neq \emptyset$, too. For that we construct the graphs $J_k = (X_k, U_k)$, $\forall k \in \mathbb{N}$, where
$X_k = \{z_1, z_2, z_3, z_4, z_5, z_6, x_1, y_1, x_2, y_2, \ldots, x_k, y_k\}$,
$U_k = \{(z_1, z_4); (z_1, z_5); (z_2, z_5); (z_2, z_6); (z_3, z_4); (z_3, z_6)\} \cup$
$\cup \{(x_i, z_4); (x_i, z_5); (x_i, z_6) \mid \forall i \in \{1, 2, \ldots, k\}\} \cup$
$\cup \{(y_i, z_1); (y_i, z_2); (y_i, z_3) \mid \forall i \in \{1, 2, \ldots, k\}\}$, Fig. 3.

By direct verification we can see that graphs J_k, $\forall k \in \mathbb{N}$ are d-convex simple and that no one vertex is dominated and respectively does not exist any pair of copies vertexes, therefore it results $\{J_k,\ \forall k \in \mathbb{N}\} \subset \mathcal{B}_2$. So we have that $\mathcal{B}_2 \neq \emptyset$.



a) b) c)

Figure 3.

Let now $G = (X,\ U)$ be an arbitrary, undirected, d-convex simple graph that contains one pair of copies vertex x_1 and x_2. From this graph we form a new graph, where we add one more vertex copy x_3 of vertexes x_1, x_2. Let us denote this graph by G^{++} (this notation is borrowed from the language C++, where i++ increases the value of i by one entity).

Lemma 3. *For any finite graph G are true the next assertions:*

1. *If $G \in \mathcal{G}$, then $G^{++} \in \mathcal{G}$;*
2. *If $G \in \mathcal{A}$, then $G^{++} \in \mathcal{A}$;*
3. *If $G \in \mathcal{B}$, then $G^{++} \in \mathcal{B}$;*

Proof. 1. Indeed, let $G \in \mathcal{G}$ be any d-convex simple graph that contains one pair of copies x_1, x_2, and x_3 is their new copy in G^{++}. Then the d-convex hull of any two nonadjacent vertexes from G will contain together with x_1, x_2 the vertex x_3, in G^{++}. The d-convex hull of vertexes x_3, y, $\forall y \in G$, $y \not\sim x_3$, will contain the same vertexes as the convex hull of vertexes x_1, y or x_2, y, because they are copies, but the d-convex hull of each of the last pair contains all vertexes of G, because G is d-convex simple. So we have showed that the d-convex hull of any pair of nonadjacent vertexes of G^{++} contains all vertexes of this graph, from which it results that G^{++} is d-convex simple: $G^{++} \in \mathcal{G}$.

2. Let $G \in \mathcal{A}$, so G is a d-convex simple graph, where any vertex is dominated by other. From the first part of this proof we have that G^{++} is d-convex simple, remains to show that G^{++} does not contain vertexes, that are not dominated. The new added vertex x_3 is dominated by their copies x_1 and x_2, all other vertexes being dominated by condition, it results $G^{++} \in \mathcal{A}$.

3. Let $G \in \mathcal{B}$, so G is a d-convex simple graph which has at least one vertex v that is not dominated and let G have a pair of copies vertexes x_1, x_2. Then from the first part of this proof we have that G^{++} is d-convex simple, where the vertex v is not dominated, because x_3 cannot dominate v as a copy of x_1, otherwise x_1 dominates v in G, too. Other new vertexes that should dominate the vertex v also do not exists, it results $G^{++} \in \mathcal{B}$.

□

So we have introduced an operation that allows us the multiplication of copies vertexes, i.e. if we have a d-convex simple graph G with a pair of copies vertexes, then we can form a new d-convex simple graph, analogical with G, but where instead of two copies vertexes we have n copies vertexes, $\forall n \in \mathbb{N}$, n is fixed. For example, by using operation of multiplication, we obtain from the graph H (Fig. 2b) a countable set of M-prime graphs and with vertexes that are not dominated: $\{H_k \,|\, k \in \mathbb{N}\}$, Fig. 4.

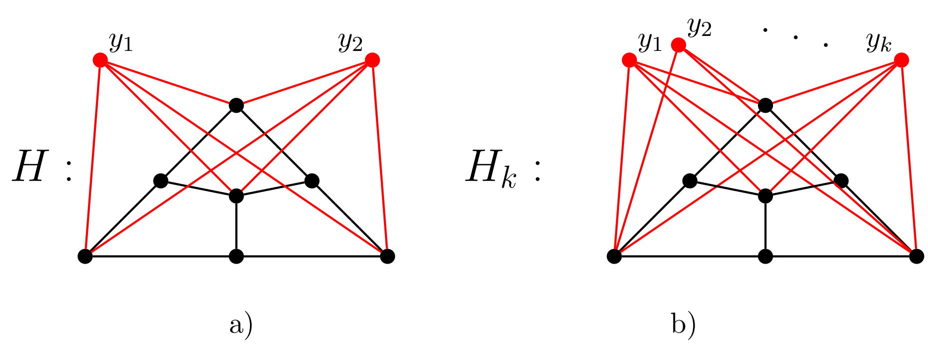


Figure 4.

If a graph G contains more than one pair of copies vertexes, then we can use the operation of multiplication over all of them, or only on some of them arbitrary, not necessarily equal numbers of times.

The reverse operation it is also true, i.e. if we have a d-convex simple graph G and x_1, x_2, x_3 are three copies vertexes of G, then the graph G^{--}, where x_3 is missing, will be also d-convex simple.

Let us now have a d-convex simple graph G and v an arbitrary vertex of it. Let us form the graph G^{++}, where we have added a copy vertex for v, which we denote v'. The next lemma is true:

Lemma 4. *If G is a d-convex simple graph then G^{++} is also d-convex simple.*

Proof. Let $G \in \mathcal{G}$ be any d-convex simple graph and v an arbitrary vertex of it. Let us form the graph G^{++}, where v has a copy v'. The d-convex hull of any two nonadjacent vertexes from G, different from v and v', will contain together with v the vertex v' in G^{++}. G is d-convex simple, we have that v is not a suspended vertex, i.e. v is adjacent to at least two nonadjacent vertexes, because G does not contain triangles, so we have that d-segment $\langle v, v' \rangle$ contains at least two nonadjacent vertexes x, x', and d-convex hull of these will contains all vertexes of G. By this way we obtain $\langle v,\ v' \rangle = X_{G^{++}}$. Let now y be a vertex from G such that $d(v,\ y) = 2$. d-Segment $\langle v,\ y \rangle$ will contain in G at least two nonadjacent vertexes w_1, w_2, which in G^{++} will be also adjacent with v', so we have $\langle v,\ y \rangle = G^{++}$. But $\langle v',\ y \rangle = \langle v,\ y \rangle = X_{G^{++}}$. We have already proved that d-convex hull of any two vertexes at distance two in G^{++} contains all vertexes of this graph, result the graph G^{++} is d-convex simple. □

The last lemma allows to use the operation of multiplication in one d-convex simple graph on any vertex we want and new graph will be also d-convex simple. So we can construct from graphs of set $\mathcal{B}_2$ the graphs, that will belong to $\mathcal{B}_1$, or even to $\mathcal{A}$, if we will duplicate all vertexes that are not dominated. We observe that the graphs H, H_k (Fig. 4) can be obtained by using the operation of multiplication on vertex y_1 in graph J_1 (Fig. 3a). In Fig. 5 we have other graphs that are derived from the graph J_1.

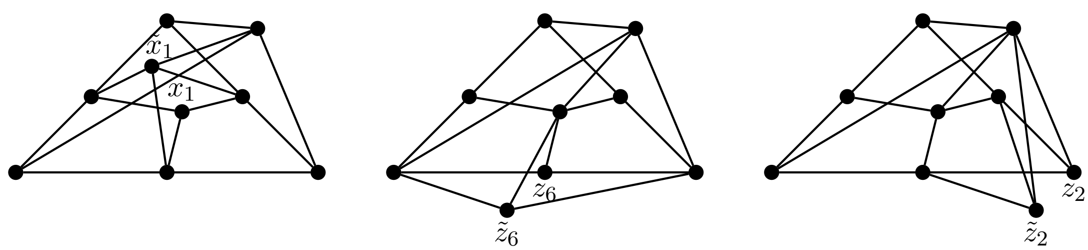


Figure 5.

Thus we are as close as it was possible to the description of d-convex simple M-prime graphs, with vertexes that are not dominated. Now let us prove the next theorem:

Theorem 6. *The next relations are true:*

$$\mathcal{A}[\mathcal{B}] = \mathcal{A}[\mathcal{B}_1] \cup \mathcal{B}_2 = \mathcal{G};$$

Proof. The equality $\mathcal{A}[\mathcal{B}] = \mathcal{A}[\mathcal{B}_1] \cup \mathcal{B}_2$ is true because the graphs of $\mathcal{B}_2$ have not pairs of copies vertexes, so they cannot participate in the M-operation and respectively cannot generate new graphs in this way.

The inclusion $\mathcal{A}[\mathcal{B}] = \mathcal{A}[\mathcal{B}_1] \cup \mathcal{B}_2 \subseteq \mathcal{G}$ is true, because we have already proved that the M-operation is algebraic on $\mathcal{G}$. Let us prove the reverse inclusion.

Let $G \in \mathcal{G}$ be any d-convex simple graph. If G has not any pair of copies vertexes then $G \in \mathcal{B}_2$. Otherwise $G \in \mathcal{A}[\mathcal{B}_1]$. So we have $\mathcal{G} \subseteq \mathcal{A}[\mathcal{B}_1] \cup \mathcal{B}_2$. □

The last theorem is very close to our goal, the goal which would be achieved if we could describe in some way all graphs from $\mathcal{B}$.

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Received November 11, 2007