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A numerical approximation of the free-surface heavy inviscid flow past a body

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Abstract. The object of this paper is to apply the Complex Variable Boundary Element Method (CVBEM) for solving the problem of the bidimensional heavy fluid flow over an immersed obstacle, of smooth boundary, situated near the free surface in order to obtain the perturbation produced by its presence and the fluid action on it. Using the complex variable, complex perturbation potential, complex perturbation velocity and the Cauchy's formula the problem is reduced to an integro-differential equation with boundary conditions. For solving the integro-differential equation a complex variable boundary elements method with linear elements is developed. We use linear boundary elements for discretize smooth curve, and free surface, in fact we approximate them with polygonal lines formed by segments, and we choose for approximating the unknown on each element a linear model that uses the nodal values of the unknown. Finite difference schemes are used for eliminating the derivatives that appear. The problem is finally reduced to a system of linear equations in terms of nodal values of the components of the velocity field. All coefficients in the mentioned system are analytically calculated. Those arising from singular integrals are evaluated using generalized Cauchy integrals. After solving the system we obtain the velocity and further the local pressure coefficient and the fluid action over the obstacle can be deduced. For evaluating the coefficients and for solving the system to which the problem is reduced, we can use a computer code.

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1 Introduction

Let us consider a uniform steady potential plane free surface flow of a heavy inviscid fluid past an arbitrary wing (obstacle) immersed in the immediate proximity of the free surface. Assuming that the boundary $\Gamma$of the wing is smooth enough to avoid the existence of some angular points (and implicitly of a Kutta type condition), we intend to set up a numerical procedure-backed by a CVBEM, for determining the perturbation induced by the presence of the obstacle (wing) and the action exerted by the fluid on this obstacle. The objective is to find the fluid velocity field and the local pressure coefficient. Using a CVBEM with linear boundary elements the problem is finally reduced to a system of linear equations. This problem is solved in [2] using Schwarz principle, without a free-surface discretization, and in paper [1] by means of linear boundary elements, but for obtaining the system's coefficients, a theorem which makes connection between the analytic function $\omega(z)$, defined by the contour integral $\omega(z)=\int\limits_{\Gamma}\frac{h(\zeta)}{\zeta-z}d\zeta$, and $\omega\prime(z)$ is used. In the herein paper other techniques are used for evaluating system's coefficients. For a better understanding, a short presentation of the problem is considered necessary, and it is made according to [2].

By splitting the velocity potential Φ into the unperturbed (uniform) stream potential and the perturbation (due to the obstacle) potential, and using dimensionless variables we have $\Phi(x,y)=x+\varphi(x,y)$, where $\varphi(x,y)$ is the perturbation potential which satisfies the Laplace equation $\Delta\varphi(x,y)=0,\ x\in(-\infty,+\infty),\ y\in(-\infty,0)$.

Assuming, at the beginning that the free surface can be approximated by the real axis Ox, by linearizing the Bernoulli's integral, the following boundary condition on free surface holds:

$$\frac{\partial^2\varphi}{\partial x^2}+k_0\frac{\partial\varphi}{\partial y}=0,\ x\in(-\infty,+\infty),\ y=0,\ (x,y)\notin\Gamma, \tag{1}$$

where $k_0=\frac{1}{Fr^2},\ Fr=\frac{U}{\sqrt{gL}}$, L and U being the characteristic length and velocity.

On the surface of the immersed wing the slip condition becomes

$$\frac{\partial\varphi}{\partial n}\mid_{\Gamma}=-n_x \tag{2}$$

where $\overline{n}(n_x,n_y)$ is the outward unit normal drawn on Γ while, on far field, $\lim\limits_{x\to\infty}\varphi(x,y)=0$.

By introducing the stream (perturbation) function $\psi(x,y)$, and by using the complex variable $z=x+iy$ and the complex (perturbation) velocity, $w=u-iv$, $(u=\frac{\partial\varphi}{\partial x},\ v=\frac{\partial\varphi}{\partial y})$, the complex (perturbation) potential $f(z)=\varphi(x,y)+i\psi(x,y)$ satisfies relations:

$$\frac{df}{dz}=\frac{\partial\varphi}{\partial x}+i\frac{\partial\psi}{\partial x}=w,\quad \operatorname{Re}\frac{d^2f}{dz^2}=\frac{\partial^2\varphi}{\partial x^2},\quad \operatorname{Im}\frac{df}{dz}=-\frac{\partial\varphi}{\partial y}.$$

Hence the previous conditions (1) and (2) become

$$\operatorname{Im}\left(i\frac{d^2f}{dz^2}-k_0\frac{df}{dz}\right)=0,\quad \text{for}\quad z=x\in R\ \ (y=0);$$

$$\operatorname{Re}\left(\frac{df}{dz}(n_x+in_y)\right)=-n_x,\quad \text{on}\quad \Gamma \tag{3}$$

By introducing the holomorphic (in the flow domain) function F, defined by:

$$F(z)=i\frac{d^2f}{dz^2}-k_0\frac{df}{dz}=i\frac{dw}{dz}-k_0w \tag{4}$$

we get:

$$\operatorname{Im}F(z)=0,\ \text{for}\ z=x\in R. \tag{5}$$

2 The Boundary Integro-Differential Equation

As $\lim_{|z|\to\infty} F(z) = 0$, the use of the Cauchy's formula for the whole domain (the lower half plane without the obstacle domain) allows us to write

$$\frac{1}{2\pi i}\int_{-\infty}^{+\infty}\frac{F(\varsigma)}{\varsigma - z}d\varsigma = F(z) - \frac{1}{2\pi i}\int_{\Gamma}\frac{F(\varsigma)}{\varsigma - z}d\varsigma \tag{6}$$

Replacing the expression of F from (4) in the above relation and using (5) we can write:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Re}\,(F(\varsigma))}{\varsigma - z}d\varsigma = -\frac{dw(z)}{dz} - ik_0 w(z) + \frac{1}{2\pi i}\int_{\Gamma}\left(k_0 i\frac{w(\varsigma)}{\varsigma - z} + \frac{w(\varsigma)}{(\varsigma - z)^2}\right)d\varsigma \tag{7}$$

3 The Discrete Equation

For solving the integro-differential equation (7) a complex variable boundary elements method with linear elements will be developed. As regards the term $\frac{dw(z)}{dz}$ an appropriate finite difference scheme will be used. Following the same steps as in [2], the border Γ is discretized by choosing a set of control points of affixes z_i, $i = \overline{1,N}$. Consequently the smooth curve Γ is approximated by a polygonal line made by segments L_j, $j = \overline{1,N}$, whose edges have the affixes z_j, z_{j+1}, $j = \overline{1,N}$, $z_{N+1} = z_1$. Using linear boundary elements (L_j) and a linear approximation for $w(z)$ of the type (see [3])

$$\widetilde{w}(\varsigma) = w(z_j)\frac{\varsigma - z_{j+1}}{z_j - z_{j+1}} + w(z_{j+1})\frac{z_j - \varsigma}{z_j - z_{j+1}}, \quad j = \overline{1,N} \tag{8}$$

(precisely all the elements with index $N+1$ are seen as having the index 1), by denoting $w(z_i) = w_i$ and by introducing the additional denotations

$$a_j(z) = \int_{L_j}\frac{\varsigma - z_{j+1}}{(z_j - z_{j+1})(\varsigma - z)^2}d\varsigma; \qquad b_{j+1}(z) = \int_{L_j}\frac{z_j - \varsigma}{(z_j - z_{j+1})(\varsigma - z)^2}d\varsigma \tag{9}$$

$$c_j(z) = \int_{L_j}\frac{k_0 i(\varsigma - z_{j+1})}{(z_j - z_{j+1})(\varsigma - z)}d\varsigma; \qquad d_{j+1}(z) = \int_{L_j}\frac{k_0 i(z_j - \varsigma)}{(z_j - z_{j+1})(\varsigma - z)}d\varsigma,$$

$$m_j = a_j + c_j, \quad n_{j+1} = b_{j+1} + d_{j+1}, \quad A_j = m_j + n_j, \; j = \overline{1,N} \tag{10}$$

equation (7) gets the form:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z}d\varsigma+\frac{dw\left(z\right)}{dz}+ik_0w\left(z\right)=\frac{1}{2\pi i}\sum_{j=1}^{N}w_jA_j \tag{11}$$

The involved integrals may be analytically evaluated, and so the above unknowns coefficients. Making the effective calculations and considering that $z_0=z_N$, we get for $j=\overline{1,N}$ the following expression for them:

$$A_j=\left[\frac{1+ik_0\left(z-z_{j+1}\right)}{z_j-z_{j+1}}\right]\ln\left(\frac{z_{j+1}-z}{z_j-z}\right)+\left[\frac{-1+ik_0\left(z_{j-1}-z\right)}{z_{j-1}-z_j}\right]\ln\left(\frac{z_j-z}{z_{j-1}-z}\right) \tag{12}$$

Now if we let $z\to z_i\in\Gamma$, $i=\overline{1,N}$, backed on the results of [4], we obtain:

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma+\frac{dw\left(z_i\right)}{dz}+ik_0w\left(z_i\right)=\frac{1}{2\pi i}\sum_{j=1}^{N}w_jA_{ij} \tag{13}$$

The two indexes point out that the limits of coefficients (11), when $z\to z_i\in\Gamma$, are considered.

Concerning the coefficients from (11), their calculation is performed by imposing effectively $z\to z_i\in\Gamma$ in the previous expressions of A_j,so in (12). Except the elements originated from the integrals calculated on segments $\Gamma_{i-1}$and Γ_i, which become singular, this implies a simple replacement of z with z_i. With regard to the coefficients coming from the singular integral, we shall use some results obtained in [5] for the evaluation of a principal value (in the Cauchy sense) of a singular integral of the type $\int_\Gamma\frac{f\left(\xi\right)}{\left(\xi-z\right)^2}d\xi$ (Γ being a closed segmentary smooth curve) and the equality $\lim_{z\to z_i}\left(z-z_i\right)\log\left(z-z_i\right)=0$ (see [6]). We get the following expressions:

$$A_{ij}=\left[\frac{1+ik_0\left(z_i-z_{j+1}\right)}{z_j-z_{j+1}}\right]\ln\left(\frac{z_{j+1}-z_i}{z_j-z_i}\right)+\left[\frac{-1+ik_0\left(z_{j-1}-z_i\right)}{z_{j-1}-z_j}\right]\ln\left(\frac{z_j-z_i}{z_{j-1}-z_i}\right),$$

$j\neq i-1,i,i+1,$

$$A_{ii}=ik_0\ln\left(\frac{z_{i+1}-z_i}{z_{i-1}-z_i}\right)+\frac{1+\ln\left|z_{i-1}-z_i\right|}{z_{i-1}-z_i}+\frac{-1+\ln\left|z_{i+1}-z_i\right|}{z_i-z_{i+1}},$$

$$A_{ii-1}=\left[\frac{-1+ik_0\left(z_{i-2}-z_i\right)}{z_{i-2}-z_{i-1}}\right]\ln\left(\frac{z_{i-1}-z_i}{z_{i-2}-z_i}\right)+\frac{1+\ln\left|z_{i-1}-z_i\right|}{z_i-z_{i-1}}$$

$$A_{ii+1}=\left[\frac{1+ik_0\left(z_i-z_{i+2}\right)}{z_{i+1}-z_{i+2}}\right]\ln\left(\frac{z_{i+2}-z_i}{z_{i+1}-z_i}\right)+\frac{1+\ln\left|z_{i+1}-z_i\right|}{z_{i+1}-z_i} \tag{14}$$

where $i,j=\overline{1,N}$, while by index $N+1$ we should understand 1, by $N+2$ we understand 2, by 0 we understand N, by -1 we understand $N-1$.

The complex velocity derivative at node i is approximated by a backward finite difference scheme, namely $\dfrac{dw\left(z_i\right)}{dz}=\dfrac{w\left(z_i\right)-w\left(z_{i-1}\right)}{z_i-z_{i-1}}$, and is replaced in (13). We obtain the following system of equations:

$$\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma+\frac{w_i-w_{i-1}}{z_i-z_{i-1}}+ik_0w_i=\frac{1}{2\pi i}\sum_{j=1}^{N}w_jA_{ij},i=\overline{1,N} \qquad (15)$$

or the equivalent form:

$$\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma=\sum_{j=1}^{N}w_j\widetilde{A}_{ij},\ \ i=\overline{1,N} \qquad (16)$$

where

$$\widetilde{A}_{ij}=-\frac{1}{2\pi k_0}A_{ij},\ \ j\neq i,\ \ j\neq i+1;\ \widetilde{A}_{ii}=-\frac{1}{2\pi k_0}\left(A_{ii}-\frac{2\pi i}{z_i-z_{i-1}}\right)-1;$$
$$\widetilde{A}_{ii-1}=-\frac{1}{2\pi k_0}\left(A_{ii-1}+\frac{2\pi i}{z_i-z_{i-1}}\right) \qquad (17)$$

By denoting with v_n,v_s the normal and the tangential, respectively, components of the perturbation velocity we can write that, on the border, $w=\left(v_n-iv_s\right)\left(n_x+in_y\right)$ while on Γ, $v_n=-n_x$, so that $w=\left(-n_x-iv_s\right)\left(n_x+in_y\right)$. Equation (16) becomes:

$$\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma.=\sum_{j=1}^{N}\left(-n_x^j-iv_s^j\right)\left(n_x^j+in_y^j\right)\widetilde{A}_{ij} \qquad (18)$$

As the perturbation vanishes at far field we can accept that

$$\frac{1}{2\pi}\int\limits_{-\infty}^{+\infty}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma=\frac{1}{2\pi}\int\limits_{a}^{b}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma \qquad (19)$$

Taking into account that on the free surface the following condition holds: $\operatorname{Re}\left(F\left(z\right)\right)=-\dfrac{1}{k_0}\dfrac{\partial^2u}{\partial x^2}-k_0u,\ z=x,$ and choosing $M+1$ equidistant nodes on it, $x_0=a,\ x_k=a+k\dfrac{b-a}{M},\ k=\overline{1,M}$, in order to obtain a discretization of the free surface into M isoparametric linear boundary elements we get:

$$\frac{1}{2\pi}\int\limits_{a}^{b}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma=\frac{1}{2\pi}\sum_{l=0}^{M-1}\int\limits_{x_l}^{x_{l+1}}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma-z_i}d\varsigma=\frac{1}{2\pi}\sum_{l=0}^{M-1}\int\limits_{x_l}^{x_{l+1}}\frac{-\frac{1}{k_0}\frac{\partial^2u}{\partial x^2}-k_0u}{x-z_i}dx \qquad (20)$$

For the isoparametric linear boundary element $[x_l, x_{l+1}]$ we have: $x = x_l + t\left(x_{l+1} - x_l\right)$, $u = u_l + t\left(u_{l+1} - u_l\right)$ $t \in [0,1]$.

Following the calculations we have

$$\int\limits_{x_l}^{x_{l+1}} \frac{-\dfrac{1}{k_0}\dfrac{\partial^2 u}{\partial x^2} - k_0 u}{x - z_i} dx = B_{li} u_l + C_{li}\left(u_{l+1} - u_l\right) = \left(B_{li} - C_{li}\right) u_l + C_{li} u_{l+1} \tag{21}$$

where

$$B_{li} = -k_0\left(x_{l+1} - x_l\right) \int\limits_0^1 \frac{dt}{x_l + t\left(x_{l+1} - x_l\right) - z_i}$$

and

$$C_{li} = -k_0\left(x_{l+1} - x_l\right) \int\limits_0^1 \frac{tdt}{x_l + t\left(x_{l+1} - x_l\right) - z_i}.$$

Concerning the integrals

$$I_0 = \int\limits_0^1 \frac{dt}{x_l + t\left(x_{l+1} - x_l\right) - z_i}$$

and

$$I_1 = \int\limits_0^1 \frac{tdt}{x_l + t\left(x_{l+1} - x_l\right) - z_i},$$

they could be expressed analytically, precisely we have

$$I_0 = \frac{1}{x_{l+1} - x_l} \ln\left(\frac{x_{l+1} - z_i}{x_l - z_i}\right), \quad I_1 = \frac{1}{x_{l+1} - x_l} - \frac{x_l - z_i}{x_{l+1} - x_l} I_0,$$

where for the complex logarithm the main branch is considered. So, coefficients that arise in (21) have expressions:

$$B_{li} = -k_0 \ln\left(\frac{x_{l+1} - z_i}{x_l - z_i}\right), \quad C_{li} = -k_0\left[1 - I_0\left(x_l - z_i\right)\right] \tag{22}$$

Finally, denoting by $B'_{li} = \dfrac{1}{2\pi}\left(B_{li} - C_{li}\right) = \dfrac{k_0}{2\pi}\left[1 - I_0\left(x_{l+1} - z_i\right)\right]$, $C'_{li} = \dfrac{1}{2\pi} C_{li} = \dfrac{-k_0}{2\pi}\left[1 - I_0\left(x_l - z_i\right)\right]$, (20) becomes:

$$\frac{1}{2\pi} \int\limits_a^b \frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma - z_i} d\varsigma = \sum_{l=0}^{M-1} \left[B'_{li} u_l + C'_{li} u_{l+1}\right] \tag{23}$$

For sake of simplicity we consider $v_s^i = v_i,\ i = \overline{1,N}$, and using the above relation, and (19) we obtain the equivalent form for system (18):

$$\sum_{l=0}^{M-1}\left[B'_{li}u_l + C'_{li}u_{l+1}\right] = \sum_{j=1}^{N}\left(-n_x^j - iv_j\right)\left(n_x^j + in_y^j\right)\widetilde{A}_{ij} \tag{24}$$

As the number of unknowns $N+M+1$ is greater than the number of equations for "closing" the system we should now perform $z \to x_k,\ k = \overline{0,M}$ in relations (11) and (12). So we get

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\operatorname{Re}\left(F\left(\varsigma\right)\right)}{\varsigma - z_i}d\varsigma + \frac{dw\left(x_k\right)}{dx} + ik_0 w\left(x_k\right) = \frac{1}{2\pi i}\sum_{j=1}^{N} w_j \widehat{A}_{kj} \tag{25}$$

where $\widehat{A}_{kj}$ are the nonsingular integrals whose exact expressions are:

$$\widehat{A}_{kj} = \left[\frac{1 + ik_0\left(x_k - z_{j+1}\right)}{z_j - z_{j+1}}\right]\ln\left(\frac{z_{j+1} - x_k}{z_j - x_k}\right) +$$

$$+\left[\frac{-1 + ik_0\left(z_{j-1} - x_k\right)}{z_{j-1} - z_j}\right]\ln\left(\frac{z_j - x_k}{z_{j-1} - x_k}\right). \tag{26}$$

Then through (24) and a forward finite difference scheme for the complex velocity derivative of first M control points on the free surface, we get:

$$\sum_{l=0}^{M-1}\left[B'_{lk}u_l + C'_{lk}u_{l+1}\right] + \frac{w\left(x_k\right) - w\left(x_{k+1}\right)}{x_k - x_{k+1}} + ik_0 w\left(x_k\right) =$$

$$= \frac{1}{2\pi i}\sum_{j=1}^{N} w_j \widehat{A}_{kj}, \quad k = \overline{0, M-1} \tag{27}$$

For $x_k = x_M$ a backward finite difference $\dfrac{dw\left(x_M\right)}{dx} = \dfrac{w\left(x_M\right) - w\left(x_{M-1}\right)}{x_M - x_{M-1}}$ is to be envisaged. Hence

$$\sum_{l=0}^{M-1}\left[B'_{lM}u_l + C'_{lM}u_{l+1}\right] + \frac{w\left(x_M\right) - w\left(x_{M-1}\right)}{x_M - x_{M-1}} + ik_0 w\left(x_M\right) = \frac{1}{2\pi i}\sum_{j=1}^{N} w_j \widehat{A}_{Mj} \tag{28}$$

where the coefficients B'_{lk} and C'_{lk} have analogous expressions with those arising in (24), the only one difference being that now, for all nonsingular integrals (i.e., when x_k is not a node of the element on which the integral is calculated), a natural logarithm of a real number is implied.

Thus,

$$B'_{li} = \frac{k_0}{2\pi}\left[1 - I_0\left(x_{l+1} - z_i\right)\right], C'_{li} = \frac{-k_0}{2\pi}\left[1 - I_0\left(x_l - z_i\right)\right], \tag{29}$$

with $I_0 = \dfrac{1}{x_{l+1} - x_l} \ln \left| \dfrac{x_{l+1} - x_k}{x_l - x_k} \right|$, for $l \neq k-1$, $l \neq k$ when $k = \overline{1, M-1}$; $l \neq 0$ when $k = 0$; $l \neq M-1$ when $k = M$. For the singular integrals, by using their finite parts, we finally get:

$$B'_{k-1k} = B'_{kk} = B'_{00} = B'_{M-1M} = \frac{k_0}{2\pi}; \quad C'_{k-1k} = C'_{kk} = C'_{00} = C'_{M-1M} = \frac{-k_0}{2\pi} \tag{30}$$

By replacing in (27) and (28) the expression of the complex velocity on the boundary, as function of the perturbation velocity components, and using the denotation s for v evaluated on the free surface (for avoiding any confusion), we can write for $k = \overline{0, M-1}$

$$\sum_{l=0}^{M-1} \left[B_{lk} u_l + C'_{lk} u_{l+1} \right] + \frac{u_k - is_k - u_{k+1} + is_{k+1}}{x_k - x_{k+1}} + ik_0 \left(u_k - is_k \right) =$$

$$= \frac{1}{2\pi i} \sum_{j=1}^{N} \left(-n_x^j - iv_j \right) \left(n_x^j + in_y^j \right) \widehat{A}_{kj}, \tag{31}$$

respectively, for $k = M$,

$$\sum_{l=0}^{M-1} \left[B_{lM} u_l + C'_{lM} u_{l+1} \right] + \frac{u_M - is_M - u_{M-1} + is_{M-1}}{x_M - x_{M-1}} + ik_0 \left(u_M - is_M \right) =$$

$$= \frac{1}{2\pi i} \sum_{j=1}^{N} \left(-n_x^j - iv_j \right) \left(n_x^j + in_y^j \right) \widehat{A}_{Mj}. \tag{32}$$

In this way we have obtained the rest of the $M+1$ equations that ensures the mathematical coherence of our mathematical problem, i.e., the solving of the system for the components of the perturbation velocity on the free surface and on the border (boundary) of the obstacle. The final system which should be solved is made by equations (24), (31) and (32).

For the outward normal components at the control points on the boundary, we also have expressions depending on points coordinates: $n_x^j = \dfrac{\operatorname{Im} ag\left(z_j - z_{j+1} \right)}{\left| z_j - z_{j+1} \right|}$; $n_y^j = \dfrac{-\operatorname{Re} al\left(z_j - z_{j+1} \right)}{\left| z_j - z_{j+1} \right|}$, and consequently all the coefficients which are present in the final system can be expressed as functions of the discretization nodes coordinates. Their calculation, system's solution and evaluation of fluid action over the body, expressed by the local pressure coefficient, can be performed by a computer, irrespective of the obstacle shape and the discretization mesh used for the boundaries.

After solving the system the problem is reduced at, it is also possible to find the shape of the unknown free surface using the velocity field and the Bernoulli relation (1). But this will be the objective of a further work.

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